Numerical Investigation of Hydrodynamic Characteristics of the Micro Irrigation Lateral

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Abstract

In this paper , the analysis of the hydraulics of micro irrigation lateral is made and the effect of the design parameters (ground slope, length of lateral and velocity of flow) on the hydrodynamic performance of this lateral pipe is numerically studied. The numerical solution for this model is made using the finite difference method which considered accurate and simple compared with other analytical and numerical methods used in literature to study micro irrigation systems.

The numerical model used in this paper is validated by comparing its results with results of another numerical methods presented in literature .The results obtained show that, the parameters studied in this paper play an important role on the hydrodynamic performance of the micro irrigation systems.

التحليل العددي للخصائص الهيدروديناميكية لمنظومات الري بالتنقيط

المستخلص

في هذا البحث تمت دراسة وتحليل هيدروليكية منظومات الري بالتنقيط وتأثير بعض العوامل التصميمية (ميلان الأرضية، طول الأنبوب، سرعة الجريان) على كفاءة هذه المنظومات. تم حل النموذج الناتج من الاشتقاق باستخدام طريقة الفرو قات المحددة والتي تعد من الطرائق العددية الدقيقة والسهلة مقارنة بالطرائق التحليلية والعددية الأخرى المستخدمة من قبل العديد من الباحثين في هذا المجال.

تم التحقق من دقة النموذج العددي المستخدم وذلك بمقارنة نتائج الحل العددي مع نتائج طرائق عددية اخرى مستخدمة من قبل احد الباحثين.من النتائج تبين لنا مدى التأثير الكبير والفعال لهذه العوامل على الاداء الهيدروديناميكي لهذه المنظومات وكذلك دقة وايجابيات طريقة الفروقات المحددة.

1. Introduction

Micro irrigation systems can deliver water and nutrients in precise amounts and at controlled frequencies directly to the application. With micro irrigation systems an extensive network of pipes is used to distribute water to emitters which discharge it in droplets, small streams or through mini-sprayers.

It is important to calculate the pressure distribution and emitter discharge correctly along the lateral. Using equations of energy and mass conservation give two partial differential equations system. These equations describe the flow in the lateral; their solution is tedious because of interdependence of the discharge and the pressure in a non linear relation. The solution of these equations cannot be completely analytic due to the empiric relation of discharge emitters and the energy loss relations. The numerical methods are often used to determine pressure and discharge in micro irrigation lateral.

There are many researchers in literature studied the micro irrigation units by different methods.

Wu and Gitlin (1975) [1] used the energy-gradient line (EGL) method to determine the lateral pressure head and flow rate profiles.

Anyoji and Wu (1987) [2] developed a statistical approach based upon the coefficients of variation of pressure head along a lateral line.

Warrick and Yitayew (1988) [3] used a nonlinear, second-order, ordinary differential equation (Differential method DM) in which the emitters are considered to be close enough for the lateral to be regarded as a homogeneous system of main tube and a longitudinal slot. In a later study Yitayew (1989) [4] presented a simplified analytical approach for the determination of total friction head losses which is an extension of the previous analytical solution.

Scaloppi and Allen (1993) [5] derived a differential approach to multiple outlet pipes with constant and continuously variable outflow.

Hathoot et al (1993) [6] used the forward-step calculation method which was applied to accurately establish the head loss caused by friction in any pipeline.

Kang and Nishiyama (1996) [7] used a finite-element scheme and polynomial lateral flow rate equation to determine pressure and outflow distributions.

Valiantzas (1998) [8] used the constant discharge method (CDM) and variable discharge method (VDM).

Vallesquino and Luque-Escamilla (2001) [9] presented an alternative approach based on the successive – approximations method (SAM) for solving lateral hydraulic problems for laminar or turbulent flow.

Valianzas (2002) [10] presented a new analytical approach for designing multi diameter irrigation laterals, in this method analytical equations were presented for the case of obtaining a general solution by direct calculation.

Lakhdar zella and Ahmed Kettab (2002) [11] used the Range Kutta and control volumes (CVM) numerical methods to analyse the hydraulics of the lateral micro irrigation.

Demir et al (2004) [12] developed model using dimensional analysis for frictional losses in drip irrigation laterals with cylindrical type in-line emitters placed at different spacing. They found that the mathematical model may be used to determine frictional losses in drip laterals.

Zella et al (2006) [13] designed a micro-irrigation system based on control volume method using the back step procedure. The design consists of exploring an economical and efficient network to deliver uniformly the input flow rate for all emitters. This program permitted the design of large complex network of thousands of emitters very quickly.

In this paper the finite difference numerical method is used to analyse the hydraulics of micro irrigation lateral and the effect of some of design parameters on the hydrodynamic performance of this type of irrigation systems.

2. Mathematical model

To derive the mathematical model of the micro irrigation lateral the continuity equation must be applied and the principle of energy conservation on the elemental control volume on the lateral between two points i and i+l as shown in Figure (1).

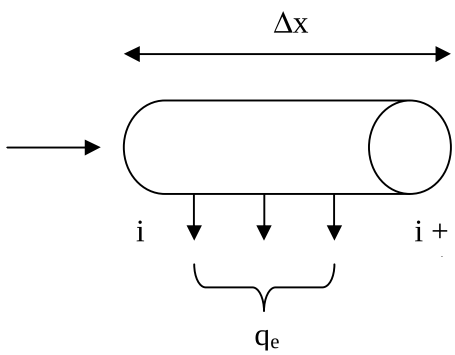


Figure (1). Control volume of micro irrigation lateral.

Continuity equation

$AV_i = A\ V_{i+l} + q_e$ (1)

Energy conservation equation

$E_i = E_{i+l} + hf$ (2)

Where:

V: flow velocity between i and i+l.

E_i and E_{i+l}: input and output energy head of water flow.

q_e: individual emitter discharge given as the follows [14]:

$q_e = C\ H^y$ (3)

In which H is the pressure head in the lateral pipe at the inlet of emitter under consideration, C is the emitter coefficient and y is the emitter exponent which characterizes the flow regime and emitter type. Values of y should range from zero for a pressure – compensation emitter to 1 for an emitter in laminar flow regime.

Applying the energy conservation equation on the same elemental control volume to give the Bernoulli's equation:

$$H_i + \frac{1}{2g}V^2{}_i + Z_i = H_{i+l} + \frac{1}{2g}V^2{}_{i+l} + Z_{i+l} + hf \quad (4)$$

Where:

hf is the head loss between i and i+l along length (Δx) and its expression is given by the Hazen-Williams formulation represented by:

$hf = a\ v^m\ \Delta x$ (5)

There are three conditions of inclination along the lateral which are zero slope (horizontal), upward and downward slope.

Zero slope condition:

Where $Z_i = Z_{i+l} = 0$

Equation (4) becomes:

$$H_i + \frac{1}{2g}V^2{}_i = H_{i+l} + \frac{1}{2g}V^2{}_{i+l} + hf \quad (6)$$

After expansion of the terms H_{i+l} and V_{i+l} in equation (6) we obtain:

$$H_i + \frac{1}{2g} V^2{}_i = H_i + \frac{\partial H_i}{\partial x} dx + \frac{1}{2g} (V^2{}_i + 2V_i \frac{\partial V_i}{\partial x} dx + (\frac{\partial V_i}{\partial x} dx)^2) + hf \tag{7}$$

Neglecting the term $(\frac{\partial V_i}{\partial x} dx)^2$ equation (7) becomes:

$$\frac{\partial H}{\partial x} dx + \frac{V}{g} \frac{\partial V}{\partial x} dx + hf = 0 \tag{8}$$

Expansion the term V_{i+1} in equation (1) gives:

$$A \frac{\partial V}{\partial x} dx + q_e = 0 \tag{9}$$

The flow regime is determined by Reynolds number.

$$\mathrm{Re} = \frac{VD}{m} \tag{10}$$

When Re > 2300, m = 1.852 and the value of a is given by the following equation when the Hazen - Williams formula is used [11].

$$a = \frac{K}{S^m A^{0.5835}} \tag{11}$$

When Re < 2300, m = 1 and a is as following:

$$a = \frac{32m}{gD^2} \tag{12}$$

Where S is the Hazen - Williams's coefficient, K is the coefficient, m is the exponent describing flow regime, g is the gravitational acceleration.

Combining equations (3), (5), (8), (9) and (12) give the following system of equations:

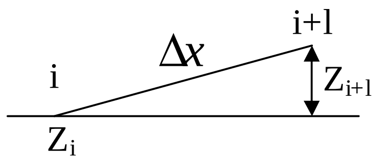
$$\frac{\partial V}{\partial x} = -\frac{C}{A\Delta x} H^y \tag{13}$$

$$\frac{\partial H}{\partial x} = -aV^m + \frac{V}{g} \frac{C}{A\Delta x} H^y \tag{14}$$

This system of equations represents the variation of pressure and velocity of flow in horizontal microirrigation lateral.

Upward slope condition:

Where $Z_{i+l} > 0$



Equation (4) becomes:

$$H_i + \frac{1}{2g}V^2{}_i = H_{i+l} + \frac{1}{2g}V^2{}_{i+l} + (Z_{i+l} - Z_i) + hf \qquad (15)$$

Let $Zi_{+l} - Z_i = \Delta Z$

$$H_i + \frac{1}{2g}V^2{}_i = H_{i+l} + \frac{1}{2g}V^2{}_{i+l} + \Delta Z + hf \qquad (16)$$

After expansion, substitution and arrangement of equation (16) give:

$$\frac{\partial H}{\partial x} = -aV^m + \frac{V}{g}\frac{C}{A\Delta x}H^y - \Delta Z \qquad (17)$$

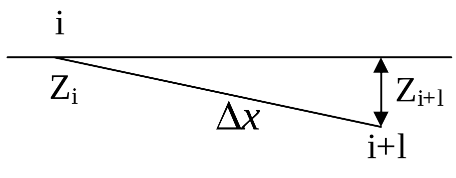
And the system of equations which represent the variation of pressure and velocity in upward slope micro irrigation lateral is:

$$\frac{\partial V}{\partial x} = -\frac{C}{A\Delta x}H^y \qquad (18)$$

$$\frac{\partial H}{\partial x} = -aV^m + \frac{V}{g}\frac{C}{A\Delta x}H^y - \Delta Z \qquad (19)$$

Downward slope condition:

Where $Z_{i+l} < 0$



Equation (4) becomes:

$$H_i + \frac{1}{2g}V^2{}_i - Z_i = H_{i+l} + \frac{1}{2g}V^2{}_{i+l} - Z_{i+l} + hf \qquad (20)$$

$$H_i + \frac{1}{2g}V^2{}_i = H_{i+l} + \frac{1}{2g}V^2{}_{i+l} - Z_{i+l} + Z_i + hf \qquad (21)$$

(22)

$$H_i + \frac{1}{2g}V^2{}_i = H_{i+l} + \frac{1}{2g}V^2{}_{i+l} - \Delta Z + hf \tag{22}$$

Note that the negative sign for Z_i in equation (20) is due to its respective elements and the value of Z_{i+l} will be Z_i for the next elements.

$$\frac{\partial H}{\partial x}dx + \frac{V}{g}\frac{\partial V}{\partial x}dx - \Delta Z + hf = 0 \tag{23}$$

After substituting the value of hf in equation (23) and arrangement, the system of equations in case of downward slope lateral becomes:

$$\frac{\partial V}{\partial x} = -\frac{C}{A\Delta x}H^y \tag{24}$$

$$\frac{\partial H}{\partial x} = -aV^m + \frac{V}{g}\frac{C}{A\Delta x}H^y + \Delta Z \tag{25}$$

The systems of equations (13), (14), (18), (19), (24) and (25) must be solved in suitable method.

Lakhdar Zella and Ahmed Kettab in ref. [11] solved simple horizontal lateral by using Range Kutta and control volumes methods. In this work the derived systems will be solved by using finite difference method and the effect of different parameters on the design of lateral will be studied.

3. Numerical solution

As mentioned before the finite difference method is used to solve the mathematical model derived in the previous section. One of the first steps to be taken in establishing a finite-difference procedure for solving a PDE is to replace the continuous problem domain by a finite-difference mesh as follows [15]: where the case of horizontal lateral is explained below as an example on the other studied cases.

The PDE system to be solved is:

$$\frac{\partial V}{\partial x} = -\frac{C}{A\Delta x}H^y$$

$$\frac{\partial H}{\partial x} = -aV^m + \frac{V}{g}\frac{C}{A\Delta x}H^y$$

The one dimensional forward difference representation is:

$$\frac{\partial u}{\partial x}\Big|i = \frac{u_{i+1} - u_i}{h} \qquad (26)$$

Substitution of equation (26) in the above PDEs gives :

$$\frac{V_{i+1} - V_i}{\Delta x} = -\frac{C}{A\Delta x}(H_i)^y$$

$$V_{i+1} = V_i - \frac{C}{A}(H_i)^y \qquad (27)$$

And $\frac{H_{i+1} - H_i}{\Delta x} = -a(V_i)^m + \frac{V_i}{g}\frac{C}{A\Delta x}(H_i)^y$

$$H_{i+1} = H_i - a\Delta x(V_i)^m + \frac{C}{gA}V_i(H_i)^y \qquad (28)$$

4. Results and discussion

4.1. Verification

To check the validity of the present numerical model used in this paper, a verification is made by solving the model presented in [11] using the present model (finite difference method) and the results are compared with both methods used in [11] which are Range Kutta and control volumes methods.

A micro irrigation lateral model presented in [11] has the following specifications: total length is 250 m and internal diameter is 15.2 mm along this lateral 50 similar emitters were placed with equal interval. The characteristics of emitter used in empirical relation (3) are C = $9.14*10^{-7}$, exponent y = 0.5, s = 150, m = 1.852, K = 5.88 and g = 9.81 m/s^s. μ = 10^6 m^2/s, initial pressure H_{max} = 30 m and initial velocity V_{max} = 1.2 m/s.

Substitution of these values in equations (27) and (28) in case of slope = 0 gives:

$V_{i+1} = V_i - 0.005\ (H_i)^{0.5}$

$H_{i+1} = H_i - 0.418\ (V_i)^{1.852} + 0.00051\ V_i\ (H_i)^{0.5}$

Figures 2 and 3 show the distributions of velocity and pressure along horizontal lateral as a comparison between finite difference method which is used in this paper and the methods of [11] (RK and CV methods). From these figures it can be seen that, there is a good agreement between the results of these three methods. Also the curve represents FDM is located between two curves of RKM and CVM which shows the accuracy of the present model.

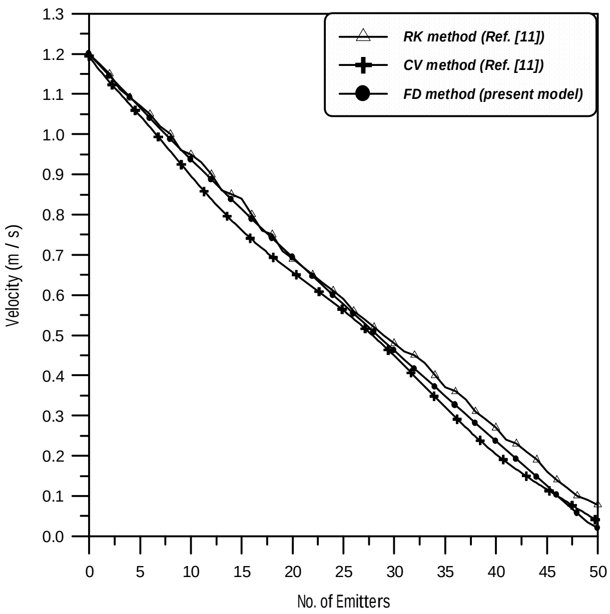


Figure (2). Velocity distribution along lateral as a comparison between different methods.

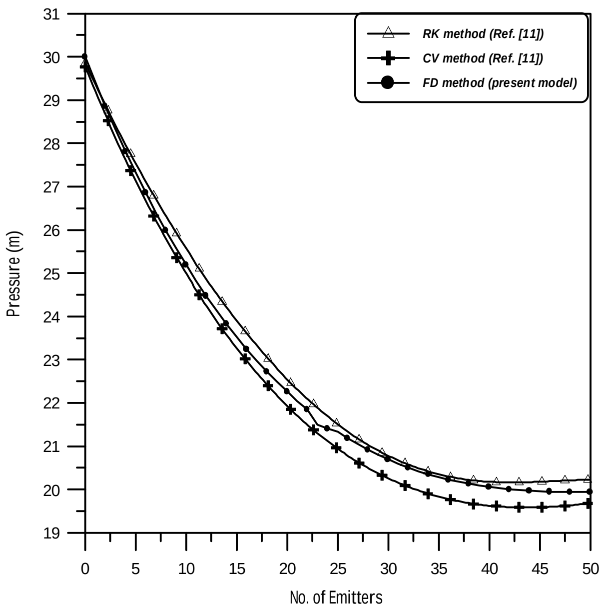


Figure (3). Pressure distribution along lateral as a comparison between different methods.

After checking the accuracy of the present model computer programs developed to calculate the distribution of pressure and velocity at the emitters along the lateral with different slope conditions. Also to calculate the effects of other parameters such as length of lateral and initial velocity.

4.2. Effect of slope

Figures 4 and 5 show the effects of lateral slope which have either a positive or negative effect on the distribution of the pressure and flow rate along lateral.

Figure (4) shows the distribution of pressure in the successive emitters along lateral for different slope conditions (horizontal, downward and upward slope). From this Figure it can be seen that, the sloped up lateral always results an increase in pressure losses along the lateral length and these losses increased with increasing the slope due to the friction and gravity effects. And the downward slope reduces the losses.

Figure 5 shows the distribution of flow rate in the successive emitters along. The lateral for different slope conditions (horizontal, downward and upward slope). From this figure one can observe that, the lateral with downward slope can deliver flow rate more than that for horizontal lateral for the same emitter. While the flow rate decreased for upward slope and the reduction in flow rate increased with increasing the slope due to the friction and gravity effects.

From the previous results shown in Figures 4 and 5 it can be seen that, the ground slope has high effect on the design of lateral of micro irrigation system since the small change in Δz (difference between the elevations of two successive nodes) caused effective influence on both the pressure head and flow rate.

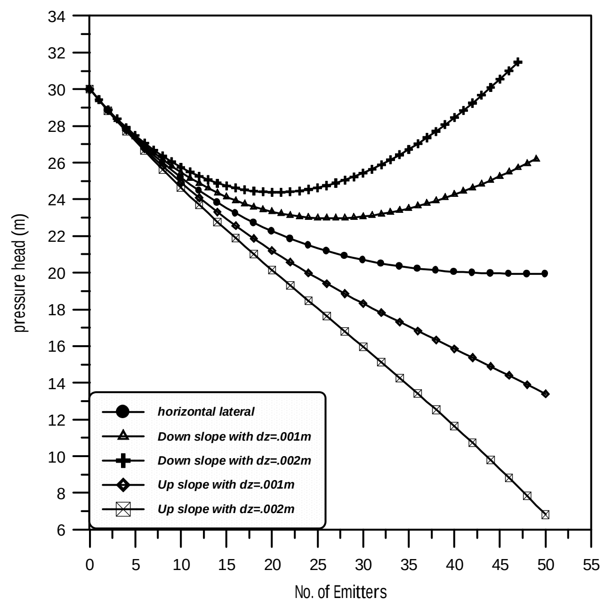


Figure (4). Effect of slope on pressure distribution along lateral.

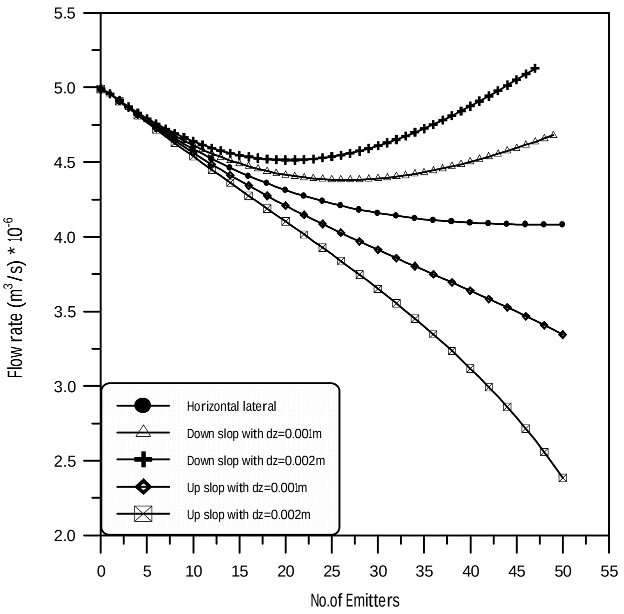


Figure (5). Effect of slope on flow rate distribution along lateral.

4.3. Effect of lateral length

Figure (6). shows the distribution of pressure head in the successive emitters along lateral for different values of length in which the pressure distribution computed for three different values of length (250 m, 350 m and 450 m) with corresponding Δx (space between two successive emitters) is (5 m, 7 m and 9 m). From this figure it can be seen that, the value of pressure decreased with increasing the length due to increase in the friction losses. Therefore this parameter must be taken into consideration in design of the micro irrigation systems.

The distribution of flow rate along the lateral for different values of length is illustrated in Figure (7). The different lengths of laterals are equipped with the same number of emitters. From this figure one can find that, the flow rate decreased along lateral for the same length due to increasing the frictional losses. Also the flow rate decreased with increase the length of lateral due to the frictional losses.

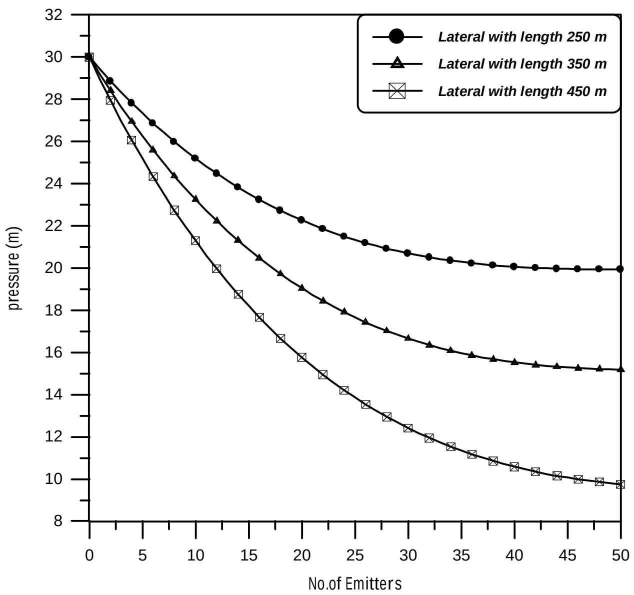


Figure (6). Effect of the lateral length on pressure distribution.

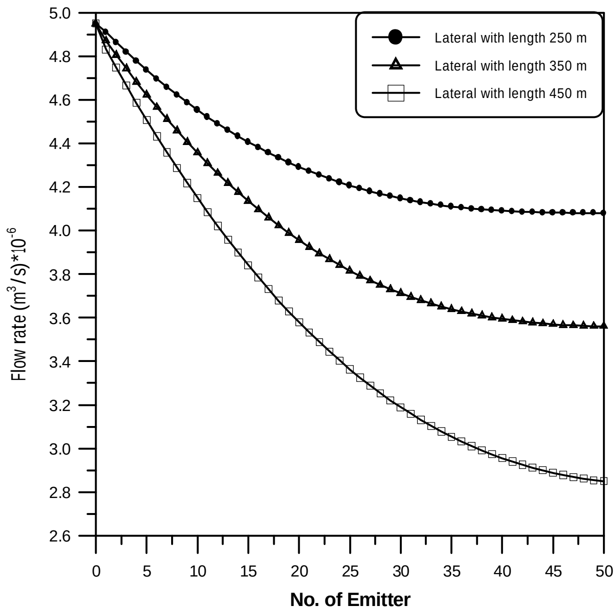


Figure (7). Effect of lateral length on flow rate distribution.

4.4. Effect of flow velocity

Figure (8). shows the effect of the velocity of source on the design of lateral in which the pressure distribution computed for three different values of velocity (0.7 m/s, 1 m/s and 1.7 m/s).

From this figure it can be seen that, the pressure head decreased along lateral for the same value of velocity due to increase in the frictional losses with length. Also the pressure head decreased with the increase in the velocity of flow due to increasing the dynamic pressure losses.

The distribution of flow rate for different emitters along lateral for different values of velocity is illustrated in figure (9). From this figure it can be seen that, the flow rate for successive emitters decreased along lateral for the same velocity which is due to the decrease in pressure head along lateral. It can be also noted that, the flow rate is decreased with increasing the velocity of flow due to the reduction in the pressure head.

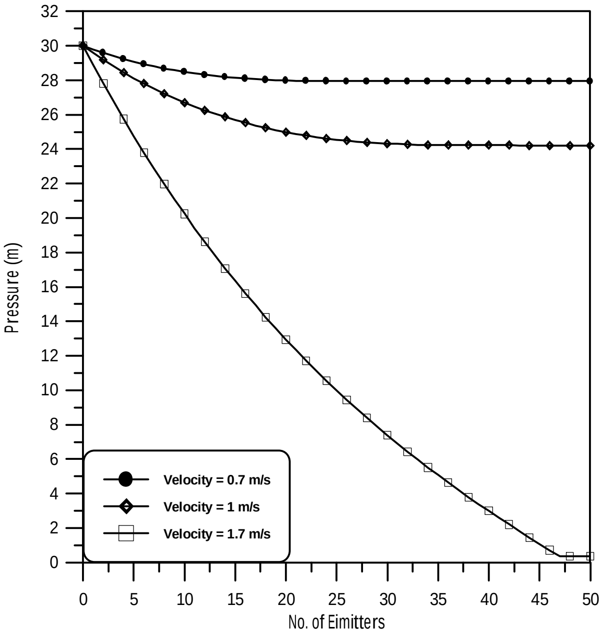
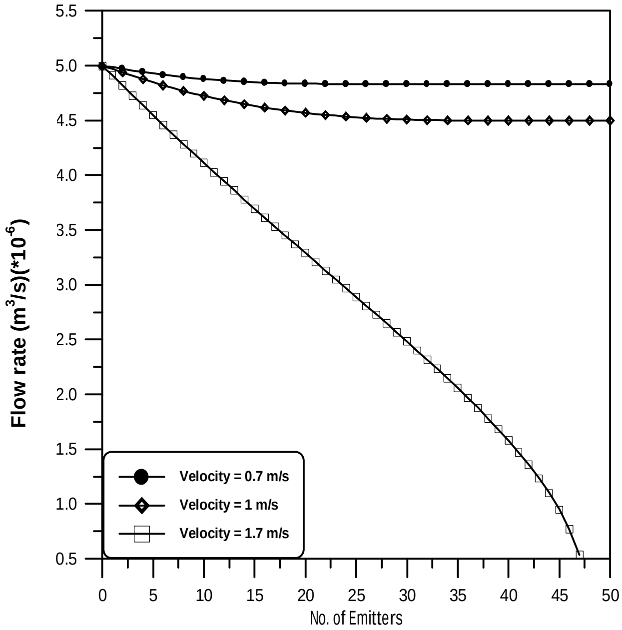


Figure (8).Effect of velocity on pressure distribution.



Figure(9). Effect of velocity on flow rate distribution.

5. Conclusions

In this paper the hydrodynamic characteristics of micro irrigation systems is numerically investigated using finite difference method and the effect of different design parameters such as (ground slope, length of lateral and velocity of flow) on the design of lateral. From the results obtained it can be concluded that:

The studied parameters have valuable effect on the hydrodynamic performance of micro irrigation systems. Since the small changes in these parameters caused large change in the distribution of the pressure and flow rate. Therefore the effect of these parameters must be taken into account in designing of micro irrigation systems and the optimum values of these parameters must be found and used in the design.

Also from the results obtained the FDM used is very accurate, efficient and simple compared with other methods used in literature and can be used accurately to design this type of irrigation systems.

6. References

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7. Nomenclature

Symbol	Description
A	Cross – sectional area, m^2
C	Emitter coefficient
D	Lateral diameter, m
E	Energy head,m
g	Gravitational acceleration, m / s^2
H	Pressure head, m
hf	Losses head ,m
Q	Discharge , m^3 / s
Re	Reynolds number
v	Velocity, m / s
y	Emitter exponent
Z	Elevation ,m
μ	Viscosity ,m^2/s