Spectrum Analysis of The Gate of Dike Structure Under Nonstationary Random Loading

Kadhim Karim Mohsen

Mechanical Engineering Department
College of Engineering
Thi-Qar University

Abstract

This paper investigates the nonstationary random excitations with a constraint on mean square value such that the response variance of a given linear system is minimized. It is also possible to incorporate the dominant input frequency into the analysis. The excitation is taken to be the product of a deterministic enveloping function and a zero mean Gaussian nonstationary random process. The power spectral density function of this process is determined such that the response variance is minimized. The numerical results are presented for multi-degree freedom system and modeled to predict the behavior of the gate of dike Structure under random water loading.

Keywords : Random loading ,seismic excitation, no stationary process .

التحليل الطيفي لبوابة السدود المائية المعرضة الى احمال عشوائية غير مستقرة

المستخلص

تم في هذا البحث در اسة الاثارة الناتجة عن الاحمال العشوائية غير المستقرة لنظام خطي مع معدل مربع القيمة للاحمال لغرض تصغير المعيارية لاستجابة النظام . كما تم استخدام مدخلات مجال التردد ضمن التحليل , مع الاخذ بنظر الاعتبار حاصل ضرب الدالة المحددة والمعدل الصفري لعملية نوع(كاوشين) العشوائية غير المستقرة حيث تم ايجاد كثافة طيف القدرة لتصغير الاستجابة المعيارية للنظام . النتائج العددية التي تم عرضها في هذا البحث للانظمة المتعددة درجات الحرية تم الاستفادة منها للتنبوء بسلوك هيكل بوابة السدود المائية المعرضة الى احمال عشوائية غير مستقرة .

1. Introduction

The analysis of flow through earth dams typically proceeds deterministically and results sometimes be quite misleading. In fact it is well known that water permeability varies randomly and spatially from point to point in space and an improved earth dam model should incorporate this variability. Stationary random field follows a lognormal distribution with prescribed mean, variance, and spatial correlation structure. The mean and variance of the total flow rate through the dam and free surface drawdown are estimated. Dikes (also known as flood gates or levees) shown in Figure(1). are used to manage or prevent water flow into specific land region, while other structures such as a dam is a barrier that impounds water or underground streams. Dikes generally serve the primary purpose of retaining waters. Hydropower and pumped-storage hydroelectricity are often used in conjunction with dikes to provide clean electricity for millions of consumers [1].

Surface topography as a nonstationary random process is often considered as a narrow bandwidth of features covering the form or shape of the surface. The study of many measurements as well as the possibility of a dominant range of features there is always an underlying random structure where undulations in surface height continue over as broad a bandwidth as the surface size will allow [2]. Many physical effects each confined to a specific waveband but no band being dominant. By invoking the central limit theorem and applying through Gaussian statistics that the variance of the height distribution of such a structure is linearly related to the length of the sample involved. In another form, the power spectral density, this relationship is shown to agree well with measurements of structures taken over many scales of size, and from throughout the physical universe. [3]. Spectral and auto correlation analysis techniques can be employed for a linear zed structure model to determine the random characteristics of structure (elongation, dynamic loads, stress). Items of interest include the peak values, RMS values, probability of exceeding a particular level or range, dominating frequencies, and further study of cumulative damage of components [4, 5].

The dynamic stiffness method applies mainly to excitations of harmonic nodal forces. For distributed loads, modal analysis is generally required. If the distributed load is adequately represented, explicit exact solutions will be found. A structure with members having distributed loads can be analyzed by two systems: one is associated with the individual members having distributed loads and the other is associated with resulting equivalent nodal forces. The required frequency functions are given for all possible cases. Contact area by taking local, weighted spatial average to account for the distributed contact [6]. Statistical properties such as power spectral density, autocorrelation function and variance of the induced spatial excitation are related to the counterparts of the original random field. It was found that the distributed contact acts like a low-pass filter whose bandwidth is governed by the contact interface and the weight function [7, 8].

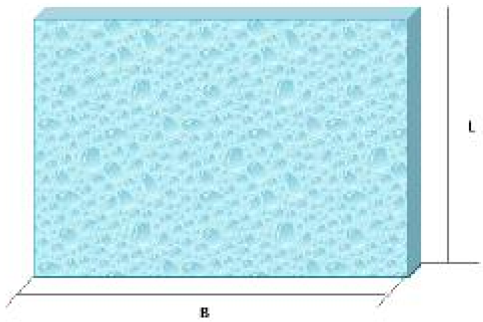

Figure (1) .Dike .

2. Analysis procedure

The input is modeled as a nonstationary random process

$$q(t) = \mathcal{U}(t)\mathcal{W}(t) \tag{1}$$

Where $\mathcal{W}(t)$ = a Gaussian nonstationary random process with zero mean and known variance; and $\mathcal{U}(t)$= a known modulating function.

$\mathcal{U}(t)$ is taken as [9]

$$\mathcal{U}(\mathrm{t}) = e^{-\beta \mathrm{t}} - e^{-\gamma \mathrm{t}} \tag{2}$$

where β and γ = parameters of modulating function.

The autocorrelation of the response of a time invariant system is given by:

$$R_x(t_1, t_2) = \int_0^{t_1}\int_0^{t_2} \mathcal{U}(\tau_1)\mathcal{U}(\tau_2)R_{\mathcal{W}}(\tau_2 - \tau_1)h(t_1 - \tau_1)h(t_2 - \tau_2)d\tau_1 d\tau_2 \tag{3}$$

Where h (t) is the impulse response function, τ time delay.

Since $\boldsymbol{\mathcal{W}}$ is a nonstationary process, its autocorrelation can be expressed as:

$$R_W(\tau) = \int_0^\infty S\,(\Omega)\,cos\Omega\tau d\Omega \tag{4}$$

where $S\,(\Omega)$ is the spectrum function of the excitation **Ω**. Thus the response variance can be written :

$$\sigma^2(\mathrm{t}) = \int_0^\infty S\,(\Omega)H(\Omega,t)d\Omega \tag{5}$$

Here H (**Ω**, t) ensembles a time-varying frequency response function.

The mean value of the spectrum excitation is defined as

$$E = \int_0^\infty S\,(\Omega)d\Omega \tag{6}$$

Using Fourier transform pair (Winer-Khintchine relation) [10] yields

$$R\,(\mathrm{\tau}) = \int_0^\infty E\,\omega(t)dt \tag{7}$$

Where $\omega(t)$ = natural frequency of the system.

Expanding Equation (7) in the series :

$$R\,(\mathrm{\tau}) = \sum_{i=1}^{\infty} E_i + \omega_i(t) \tag{8}$$

A particular solution is obtained by expanding the real function $S\,(\Omega)$ in the series

$$\sigma(S) = \sum_{i=1}^{\infty} A_i\phi_i\,(\Omega) \tag{9}$$

Where $\sigma(S)$ = the standard deviation of spectrum.

$\phi_i(i = 1,2,\ldots)$ = set of orthonormal functions such that

$$\int_0^\infty \phi_i\phi_j\,d\Omega = 0 \quad ; i \neq j \tag{10a}$$

$$\int_0^\infty \phi_i\phi_j\,d\Omega = 1 \quad ; i = j \tag{10b}$$

After using Lagrangian multiplier $\Pi(t)$, and combining Eq.8 with Eq. 9 for minimization

$$L[A_1, A_2, A_3 \dots, \Pi(t)] = \int_0^\infty \sum_{i=1}^\infty (\phi_i \phi_j)^2 H(\Omega, t) d\Omega - \omega(t) \sum A_i^2 - E \qquad (11)$$

For $\frac{\partial L}{\partial A_i} = 0 \ (i = 1,2, \dots) \ and \ \frac{\partial L}{\partial \Pi} = 0$ gives

$$\sum_{i=1}^\infty A_i f_{ij}(t) - \omega(t) A_i^2 = 0; \qquad (j = 1,2, \dots .) \qquad (12)$$

Since $f_{ij}(t)$ is a time function

$$f_{ij}(t) = \int_0^\infty (\phi_i \phi_j)^2 H(\Omega, t) d\Omega \qquad (13)$$

$$\sum_{i=1}^\infty A_i^2 = E \qquad (14)$$

This is an algebraic eigenvalue problem that can be solved using standard techniques. Thus one can get the eigenvalues $\omega_i(t)$ (i = 1,2, …) and the corresponding eigenvectors (A_{1i}, A_{2i},…) with the normalization condition of Eq. 12. Further, substitution of Eq, 12 in Eq. 5 and rearrangement leads to

$$\sigma^2(t) = \omega(t) E \qquad (15)$$

This shows that while it is possible to get as many solutions as the number of terms in the expansion for the power spectral density function PSD, it is the smallest eigenvalue and the corresponding eigenvector that leads to the lowest response variance. This is true for every time instant t. For finding the critical excitation in a given interval of time, the above equations has to be repeated for every (i), the excitation leading to the minimum response is taken as the desired solution.

Generalized model

General equation of motion for time invariant system

$$\ddot{q}_i + 2\zeta_i \omega_i \dot{q}_i + \omega_i^2 q = \mathcal{U}(t) \mathcal{W}(t) \qquad (16)$$

The displacement response variance of the ith mass of a multi degrees of freedom system system under the input of Eq. 1 is given by

$$\sigma_i^2(x,\Omega,t) = \int_0^{\infty} S\,(x,\Omega,t)H_i(x,\Omega,t)d\Omega \tag{17}$$

For complex frequency response

$$H_i(x,\Omega,t) = \iint_{0\,0}^{t_1 t_2} \sum_{k=1}^{n}\sum_{i=1}^{n} \Gamma_j\Gamma_k\phi_{ij}\phi_{ik}\mathcal{U}(\tau_1)\mathcal{U}(\tau_2)h_k(t_1-\tau_1)\,h_k(t_2-\tau_2)cos\Omega(\tau_2-\tau_1)d\tau_1 d\tau_2 \tag{18}$$

$$h_k(t) = \omega_{dk}^{-1}e^{(-\zeta_k\omega_k t)sin\omega_{dk}t} \tag{19}$$

$$\omega_{dk} = \omega_k\sqrt{1-\eta_k^2} \tag{20}$$

ω_{dk}=damped natural frequencies of the system, η_k =coefficients of viscous damping, Γ_j and ϕ_{ij}= the modal participation factors and mode shapes, respectively. The critical PSD function can be computed again as in the case of a SDOF system. However, here two important points have to be noted. First, for every level i one can get a different critical PSD function (S). Second, the critical PSD would vary depending on the response variable considered. Once the response variable is selected as to velocity, displacement, bending moment, shear etc., the computations are straightforward .The determination of the critical random input to the gate of the dike is considered as a model for the present paper. It is possible to solve the present problem in the time domain also. This involves the minimization of the response envelope in a given interval. The response envelope of response of the system governed by Eq. 16 is :

$$R(t) = 2\int_o^t [\mathcal{U}(t)\mathcal{W}(t)\dot{q}(t) - 2\eta\omega\dot{y}(t)]\dot{q}(t)dt \tag{21}$$

$\mathcal{W}(t)$ being a nonstationary random process, it can be expanded in a series as;

$$\mathcal{W}(t) = \sum_{i=0}^{n} D_i\,sin(\lambda_i t - \psi_i) \tag{22}$$

Here D_i′s are deterministic constants and ψ_i′s are independent random phase angles distributed uniformly in the interval (-π,π). Substituting Eq. 22 into Eq.21 and after some manipulation, one gets :

$$R(t) = \sum_{k=1}^{n}\sum_{i=1}^{n} D_{ij}D_{ik}f_{ik}(t) \tag{23}$$

Here $f_{ij}(t)$ is a known function of time. The RMS constraint on the input can be expressed in terms of D_i's as

$$E = \sum_{i=1}^{\infty} D_i^2 \tag{24}$$

By substituting equations (18-24) into equation (17) and completing the integration by residue to obtain the variance of the gate of the dike :

$$\sigma_i^2(x,\Omega,t) = \sum_{j=1}^{n}\sum_{k=1}^{n}\sum_{i=1}^{n} \Gamma_j\Gamma_k\phi_{ij}\phi_{ik}\mathcal{U}(\tau_1)\mathcal{U}(\tau_2)D_{ij}D_{ik}sin\left(\lambda_i t - \phi_{ij}^2\phi_{ik}^2\right) \tag{25}$$

The angular natural frequencies λ_i and Eigen functions are :

$$\lambda_i = \left(\frac{\mathcal{Z}_i}{\ell}\right)\sqrt{\frac{G}{\rho}} \tag{26}$$

$$\phi_{ij}(x) = \frac{2J_o\left(\frac{\mathcal{Z}_i(x)}{\ell}\right)}{\mathcal{Z}_i J_1(\mathcal{Z}_i)} \tag{27}$$

Here Jo and J_1 are the Bessel's function of the first kind. $\mathcal{Z}_i'$s refers to the zeros of Jo. The variance of the gate of the dike relative to the random water loading .

3. Results and discussion

A gate of water dike $\ell = 44.6\,m$ in height with a rectangular cross section under nonstationary random loading shown in Figure (2). is considered and subjected to random vibration of water. It is required to find the critical outputs such that the lateral displacement variance. The material properties of the dike are taken as density ρ=3X10^5 kN/m^3; viscous damping coefficient η=0.2; and shear modulus G= 2.92X10^6 kN/m^2.

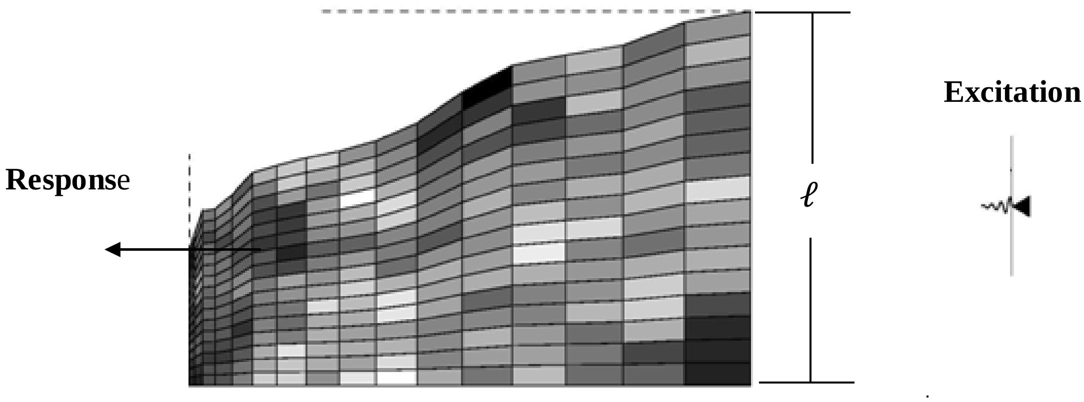

Figure (2). Systematic representation of the gate of the dike .

The numerical values of the critical outputs variance of the gate of dike can be tabulated as shown in Table (1).

Table (1). Critical outputs variance for the gate of dike .

index	λ_i/E (sec')	$\mathcal{Z}_i$	$\Gamma_j\Gamma_k$	$\sigma^2(x,\Omega,t)$
1	0.6032 x 10^{-4}	2.4048	0.85	9.6
2	0.7321 x 10^{-4}	5.5201	1.01	1.03
3	0.3216 x 10^{-4}	8.6537	1.65	0.56
4	0.2444 x 10^{-4}	11.7915	1.99	0.57
5	0.2301 x 10^{-4}	14.5032	2.01	0.5
6	0.2202 x 10^{-4}	14.8476	3.66	0.46
7	0.2121 x 10^{-4}	15.7654	4.16	0.14

The modal solution natural frequencies and mode shapes are needed to calculate the spectrum solution by applying nonstationary random excitation shown in Figure (3). The dynamic characteristics of the structure, such as the standard deviation of response are computed as shown in Figure (4).

The power spectral density shown in Figure (5). was evaluated in order to find the displacement response variance of dikes gate. Nastratn program version 4.4 was used to analyze the mode shapes of the gate of the dike shown in Figures (6-9) to obtain the natural frequencies which can be used to evaluate the variance of the response of the gate Figure (10)

The advantage of the solution in time domain is at all the desired frequencies can be included explicitly in the input. The details of this time domain solution are being currently studied in the paper. In the present method it should be noted that one does not arrive at a unique time history of excitation, but instead an ensemble of time functions forming a stochastic process is obtained.

Critical excitations as developed here are by definition system dependent. This would lead to different critical inputs for different types of dike's gate structures. This naturally is a limitation. To circumvent this difficulty one can find the critical input for the most important structure and use this to arrive at the critical excitation that is specification dependent, but is independent of the structures to be built.

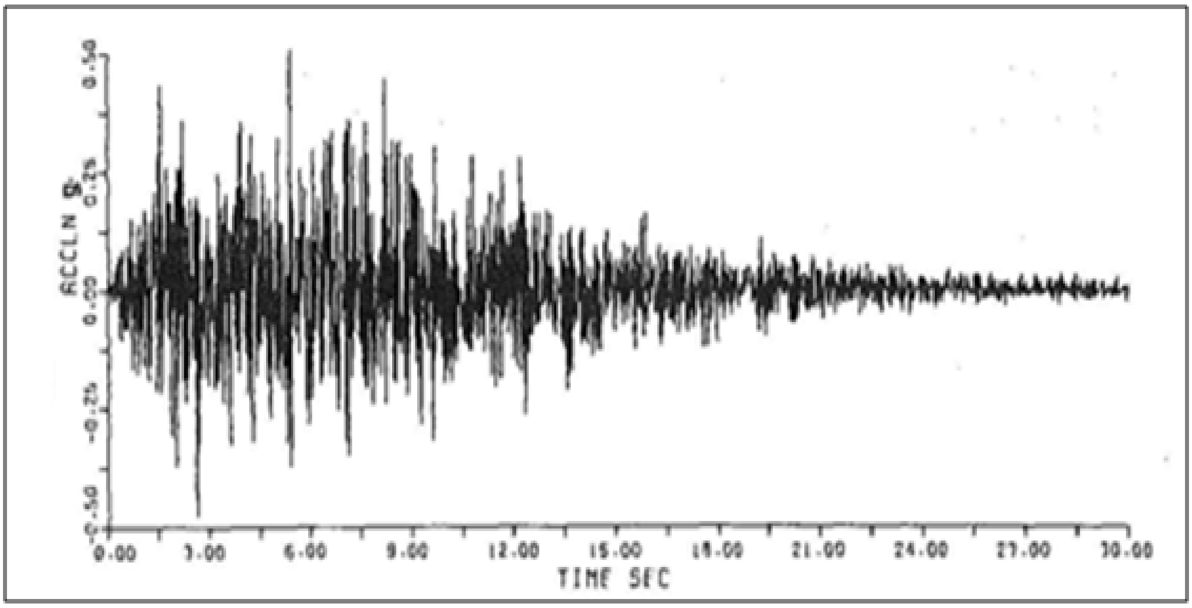

Figure (3) . Nonstationary Gaussian random excitation.

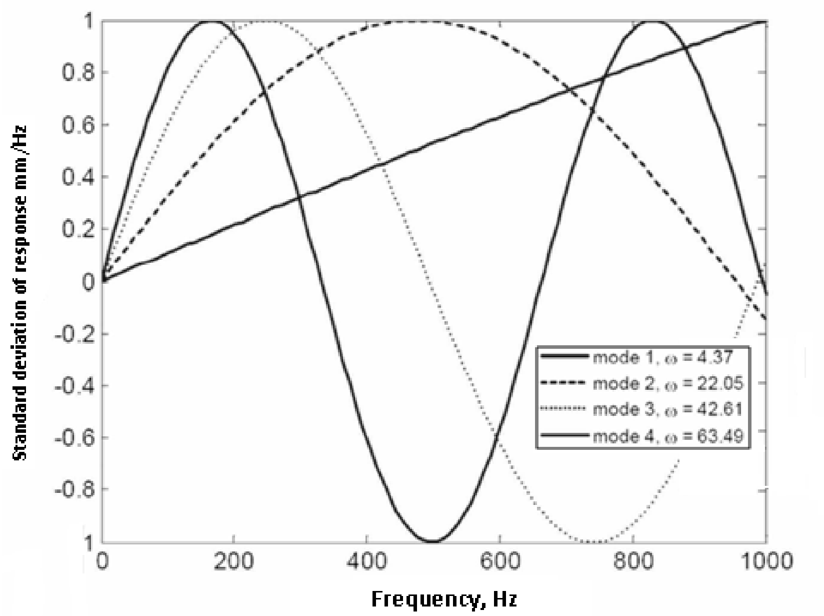

Figure (4). Standard deviation of response for the four modes of the gate of dike.

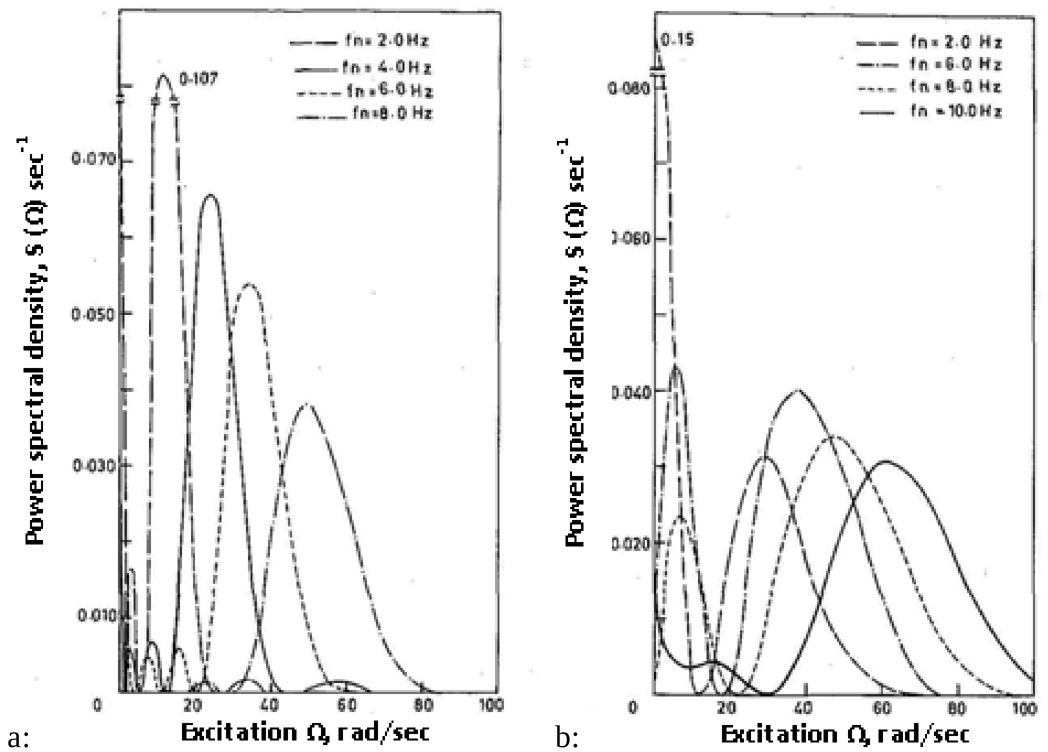

Figure (5). Critical spectrum functions for different natural frequencies of the gate of the dike (a) In terms of exponentials. (b) In terms of polynomials .

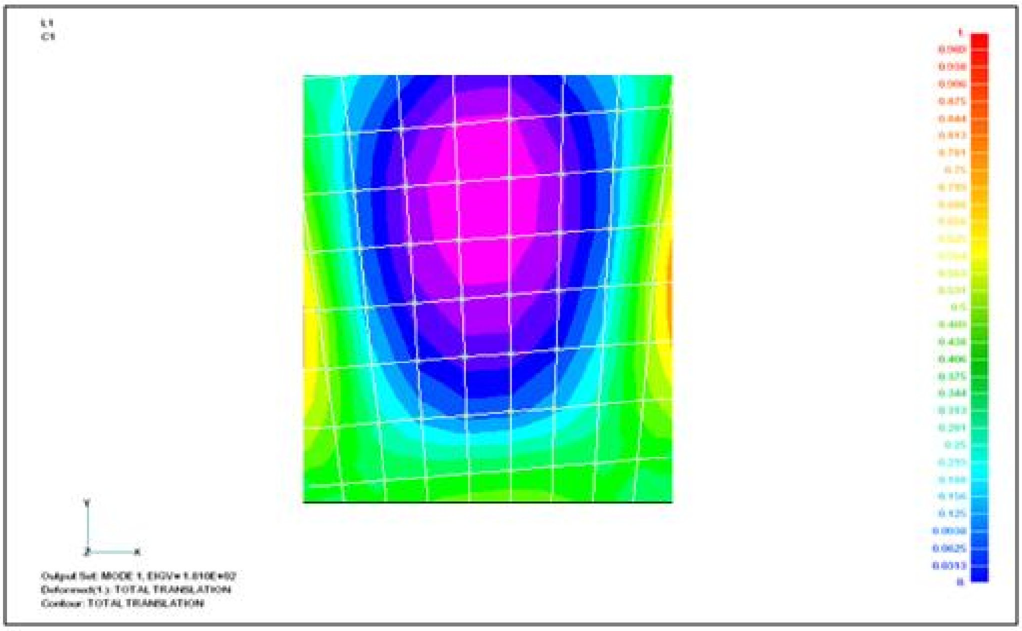

Figure (6). First mode shapes of the gate of the dike .

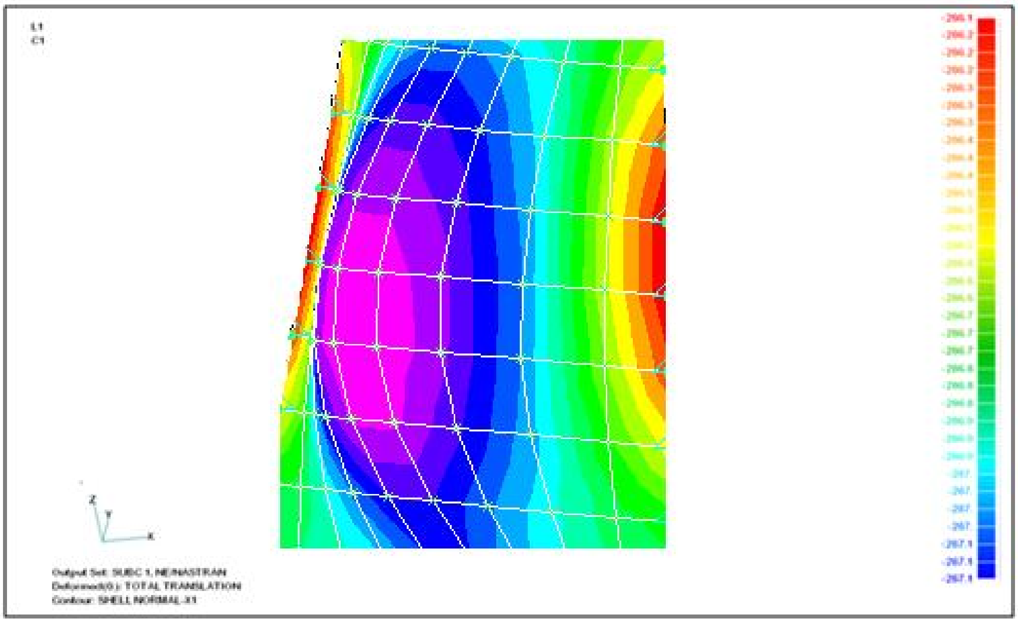

Figure (7) . Second mode shapes of the gate of the dike .

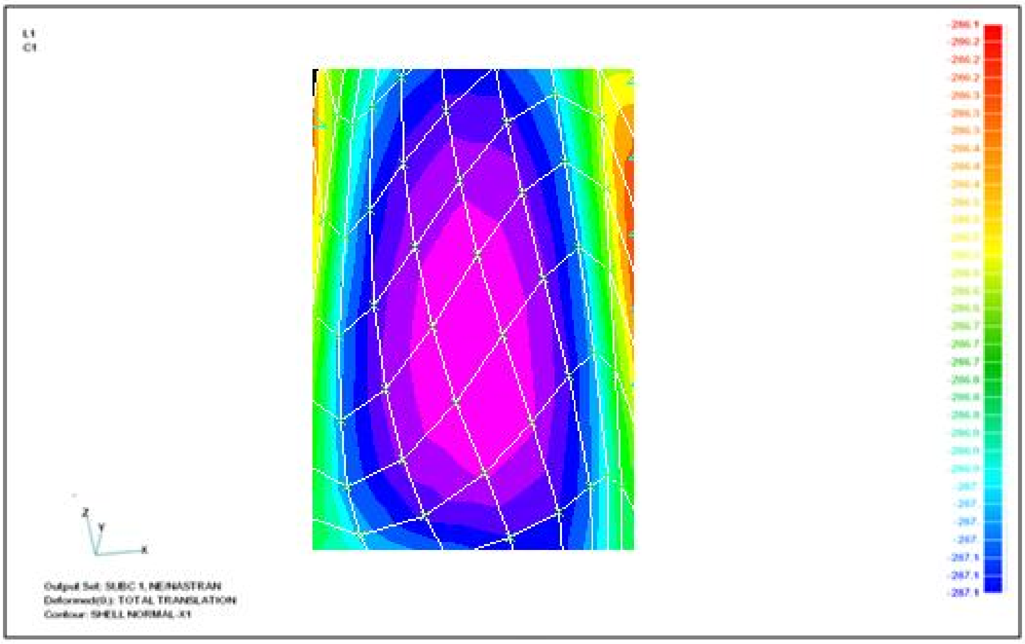

Figure (8). Third mode shapes of the gate of the dike .

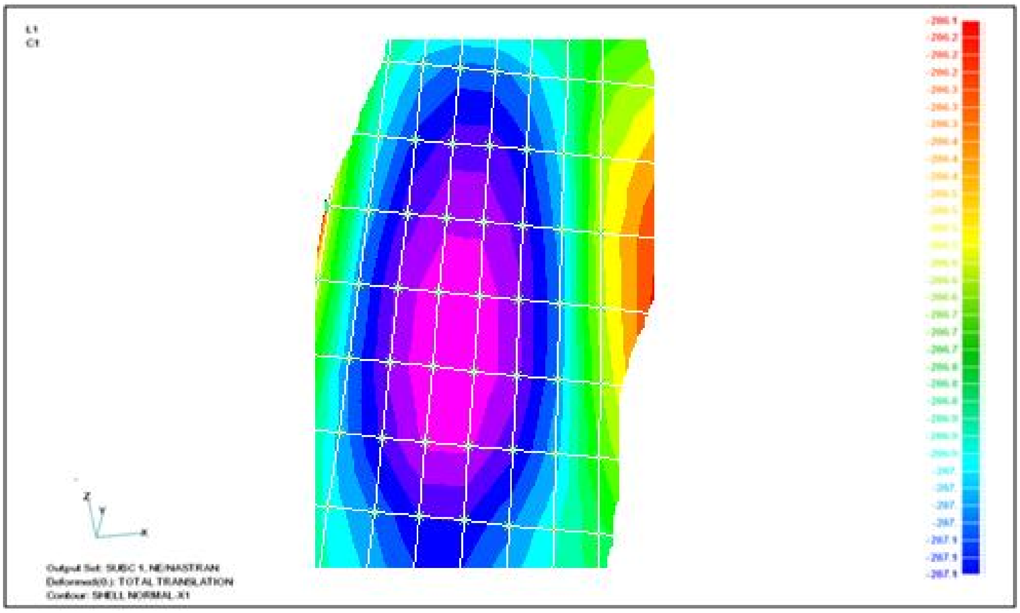

Figure (9). Fourth mode shapes of the gate of the dike.

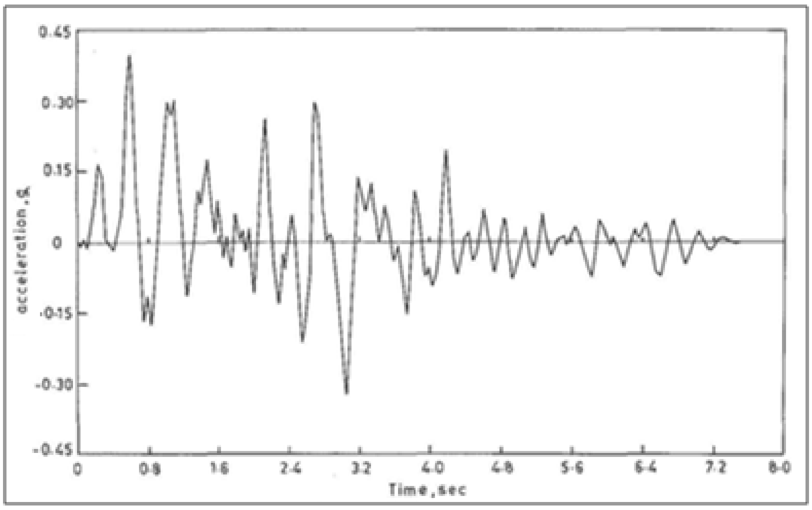

Figure (10). Displacement variance of critical excitation.

4. Summary

This study outlines a method to obtain the variance of the dike's gate at the critical random excitation for a given linear system. For purposes of application in engineering, the input is taken as an unknown nonstationary process modulated by a known enveloping function. The mean square value of the random process is required to be known. A frequency domain solution is presented for finding the critical power spectral density function of the nonstationary random input. The procedure is illustrated with the example of a gate of a dike, it is fairly obvious that the critical power spectral density function should peak near the resonant- frequency. However, with heavily damped systems and with multi degree systems, the structure of the input power spectral density function is less obvious. The numerical results obtained show that the sense of criticality is not too severe both in the input and in the response variance. Thus one can expect realistic peak excitation and peak responses from the present solution when used in the random analysis of important structures and equipment.

Further modification of the method restricts the class of allowable inputs and to minimize damage variables other than the response variance is presently under investigation which will be very useful to investigate the danger of cumulative damage in Al Mosul dike (North of Iraq).

5. References

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6. Notation

The following symbols are used in this paper:

A_i, C_i = coefficients

E= variance of w (t), m^2

λ_i = system natural frequency, Hz

G = modulus of rigidity, KN/m^2

g = acceleration due to gravity

$H(x,\Omega,t)$ = frequency response function h = impulse response function

$f_{ij}(t)$ = function of time

i,j,k = indices

J_n = Bessel's function of first kind nth order

L = Lagrangian

$\boldsymbol{\ell}$ = height of the gate of the dike

m,n = index

$\boldsymbol{\Gamma}$ = modal participation factor

$\boldsymbol{\phi}_{ij}$ = orthonormal function

R(t) = envelope of response

R_w = autocorrelation functions

S (Ω) = power spectral density function

$\mathcal{U}(t)$ = modulating function

$\mathcal{W}(t)$ = Gaussian stationary random process

q(t) = input random process

$\boldsymbol{\gamma},\boldsymbol{\beta}$ = parameters in $\mathcal{U}(t)$

$\boldsymbol{\eta},\boldsymbol{\eta}_k$ = coefficient of viscous damping

$\boldsymbol{\psi}_{ij}$ = phase angles

$\boldsymbol{\omega}_i$ = angular frequency rad/sec

$\boldsymbol{\Pi}$ = Lagrangian multiplier

$\boldsymbol{\rho}$ = density kg/m^3

$\boldsymbol{\sigma}^2$ = variance of response

$\boldsymbol{\phi}_{ij}$ = mode shapes

$\boldsymbol{\phi}_n$ = orthonormal functions

$\boldsymbol{\tau},\boldsymbol{\tau}_1,\boldsymbol{\tau}_2$ = dummy variables