Modeling of the Dissolved Oxygen Profile in Marsh Water by Numerical Solution of ADE Using Finite Difference Method

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Abstract

To evaluate the environmental impact of pollution, mathematical models play a major role in predicting the pollution level in the regions under consideration. Computer models are becoming increasingly important tools in every environmental management aspects. A general one dimensional water quality model (Advection-diffusion Equation ADE) has been developed the dissolved oxygen in Abu-Zirig marsh water. The general one-dimensional model equation was solved using the numerical solution by finite difference discritization and procedure was prepared by writing a computer program in MATLAB program (version 7.6).

Keywords : Finite difference , water pollution , dissolved oxygen.

المستخلص

لتقييم الأثر البيئي من التلوث ، والنماذج الرياضية تلعب دورا رئيسيا في التنبؤ مستوى التلوث في المناطق التي هي قيد الدراسة. اصبحت النماذج الحاسوبية أدوات هامة على نحو متزايد في كل جوانب الإدارة البيئية. تم في هذا البحث نمذجة رياضية لنوعية المياه ذات البعد الواحد (معادلة التدفق والانتشار) للمكون البيئي الأوكسجين الذائب في مياه هور ابوزرك. تم حل المعادلة النموذجية الرياضية عدديا باستخدام طريقة Finite Difference بالاستعانة ببرنامج كومبيوتري (Matlab v . 7.6).

1. Introduction

Marshes of Iraq are located in the Southern part of the country. They cover an area of about 20000 km^2. Abu Zirig marsh is located in the south of Iraq, at the tail end of Gharraf River southerly of Al Islah District in north of Nassiriah Governorate. This marsh is located in the west of the marshes in Iraq in longitude E 46° 36’ 33.0” and latitude N 31° 09’ 54.9” in north of the marsh, as shown in Figure(1).

The two main towns around the marsh are the cities of Al Islah (pop. 9,000) in the north and Al Fuhud (pop. 18,000) in the south. Scattered villages of fishermen are located all along the embankments that surround the marsh. [1]

Site Location:	Lat (**N**) 31° 09' 54.9", Lon (**E**) 46° 36' 33.0"

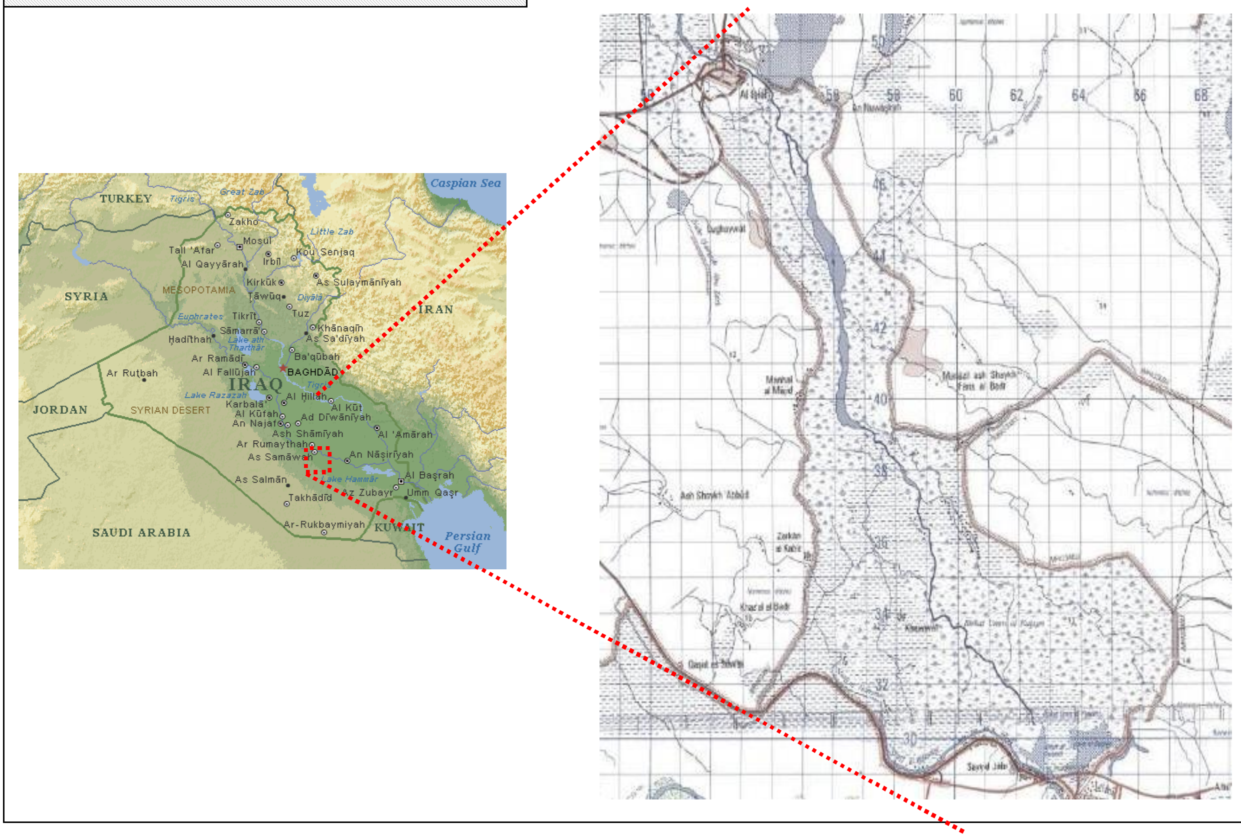

Figure (1). The location of Abu Zirig marsh in Iraq. [5]

Most of the researchers studied the water quality parameters in Iraqi marshes, but the studies about the modeling of pollutant transport in the marsh water are limited. The Abu Zirig marsh water is used for a several purposes mainly as potable water and for irrigation, The major sources of Abu Zirig marsh water pollution are industrial effluents released through shat-Algharaf and municipal wastewater.

The marsh is located in a natural depression. A series of embankments, built in 1920, protects the agricultural farms around the perimeter of the natural depression that forms the deep marsh of Abu Zirig. The main supply of water to the marsh is through Shatt Abu Lihia and the channel of this river runs through the marsh until it dissipates at the tail end into the larger marsh (Central Marsh). This marsh re-flooded in June 2003 as a result of the direct action of the local population. The area is about 120 km^2, and recovery is progressing very well, with reeds growing higher than 2 meters. Water flow into the marsh has been regulated in the Gharraf River. As a result of regulation of Gharraf River, the marsh remains in a very healthy state, despite the oncoming of summer months.[2]

Abu Zirig marsh is about 3% of all marshes in Iraq, and dried in 1991 with all the central marshes. This marsh has a single inlet in the north (Gharraf River) and 20 small outlets in the south. These outlets discharge to the central marsh (Qurnah Marsh) between Tigris and Euphrates [3]. This marsh is divided into two zones ,the first zone is the upper zone, and other zone is the lower zone, which are divided by road, as shown in Figure (2). This road has pipes and broken areas in 6 points collection between them [4].

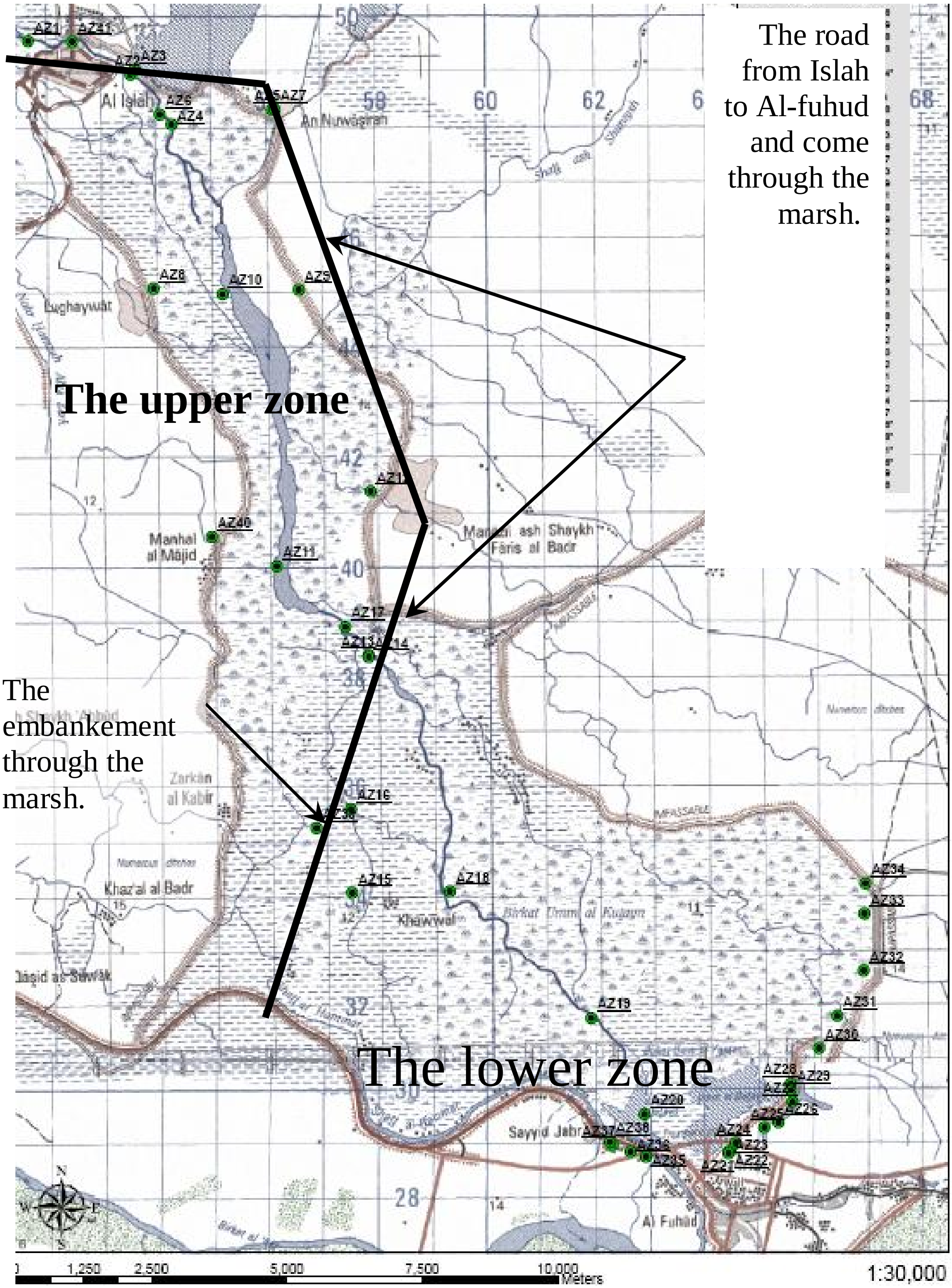

Figure (2). The upper & lower zone in Abu Zirig marshes.

3. Previous studies

Hassan F. M. (1988) studied the physical and chemical properties of Hammar marsh and their effects on the plants. This study covered the water temperature, pH, and electrical conductivity in Hammar Marsh seasonally for one year. He found that water temperature ranges from 9.9 to 34.3°C, the pH ranges from 7.2 to 8.7 and conductivity ranges from 1730 to 7100 μS/cm.

Hussein N. A. (1994) studied the main marshes, and made many measurements like air temperature, water temperature, salinity, pH, conductivity, and dissolved oxygen with some chemical, physical, and biological parameters. All these studies were seasonally in all the marshes in south of Iraq. These researches indicate that salinity increased to 6.250 ppt (parts per thousand) in 1991 depending on the inlet water from Tigris and Euphrates and there is more than the salinity in the northern part of the rivers. The D.O. depends on the plants and is less than 4 mg/l in the closed area.

Schaffranek, R.W., and Lai, C., (1996), used the friction-slope term in the unsteady open-channel flow equations which were examined using two numerical models based on different formulations of the governing equations and employing different solution methods. Results of numerical experiments illustrate that a given model can respond inconsistently for the identical resistance-coefficient value under different types and combinations of boundary conditions. Discussion of qualitative considerations and quantitative experimental results provides insight into the proper treatment, evaluation, and significance of the friction-slope term, thereby offering practical guidelines for model implementation and calibration.

Chapra (1997) classifies water quality models as empirical and mechanistic models. The empirical models are based on an inductive or data-based approach. It involves obtaining set of model input data from many systems that are similar to the water system going to be modeled (Chapra, 1997). The mechanistic models are based on a deductive or theoretical approach. The mechanistic model involves the use of theoretical relationships or organizing principles.

IMET and IF-2 (2004) calculate the area of water surfaces of all big marshes. The area of water surfaces of the Central Marsh is about 3000 km². 700 km² of the Central Marsh are natural lakes with a flow rate of about 400 m³/s. with an average depth of about 1.75 m and salinity ranging from 0.2 ppt to 1.5 ppt. The area of water surface of Hawaizah Marsh is about 3500 km² and 650 km² of Hawaizah Marsh is permanent marsh with average depth of 2.5 m and salinity ranging from 0.3 ppt to 2 ppt. At flood season, the area of water surface of

Hammar Marsh reaches 3000 km² and the total storage of 5 BCM (billion cubic meters). At dry season, the area of water surface of Hammar Marsh is 600 km² and the total storage capacity is approximately of 0.18 BCM, with a high salinity of about 0.5 ppt to 10 ppt.

4. Data collection

4.1 Hydrological balance(water –budget)

Discharge: The marsh, for this purpose has been divided into two sectors namely the Upper zone between Islah and Said Yousha'a road embankment and the Lower zone between Said Yousha'a and Al Fuhud Figure (3). Discharge inlet to the upper zone in Islah Breach is computed and output discharge in Al- Subahaa and Abu Al-jarry was also estimated the output discharge is the same input discharge to the lower zone of Abu Zirig marsh output of the lower zone and marsh also computed, It should be noted that when water levels are high, the culverts built under the embankment suffer from piping and frequent road cuts result this past flood season from the high water flows.

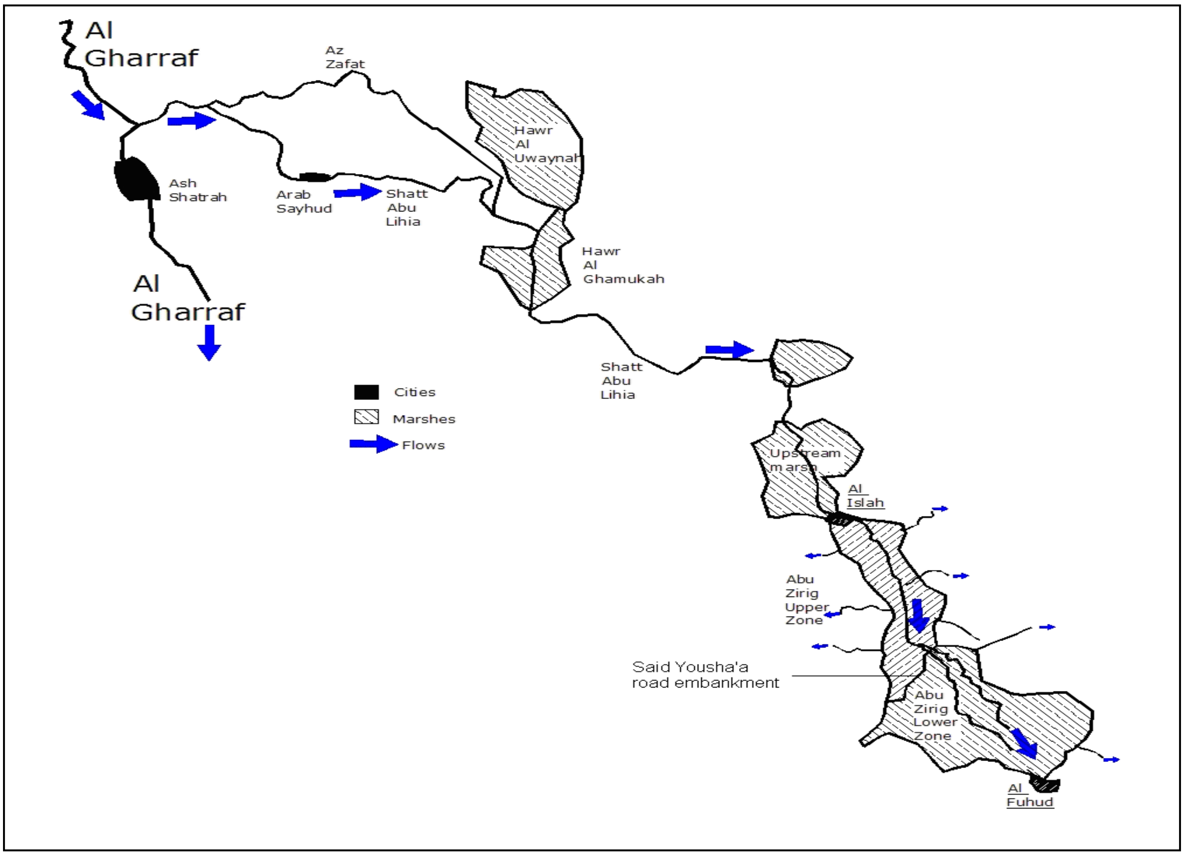

Figure (3). The hydrologic network and the road halves Abu Zirig marsh to the upper and lower part.

Discharge is calculated upstream Islah bridge in Abu Lihia River at the tail end of Gharraf River before interring the marsh, the discharge is calculated at this location by a stage measure which is readily available to measure the water surface elevation (it is attached to the bridge) and so as to be able to account for the water losses between inlet area and the main entrance to the marsh.The values of discharge are listed in table (1) from April 2004 to March 2005 [7] , average flow through this location is 32.485 m^3/s.

The second location for computing discharge is the Inlet area at the Islah Breach this is the main inlet to the marsh. Water entering this section should be the one measured at upstream Islah bridge minus the one exiting at Islah Channel. In reality, it is not always possible to balance this simple equation (possibly due to error in the measurements as well as uncounted outlets or losses). The average flow through this location is 25 m^3/sec. It also should be noted that water fluctuation during the months is not directly related to water availability in the rest of the country (Rzoska J. at.el 1980)[9].

It has been observed that, when water discharge exceeds 25-30 m^3/s at the Islah Bridge, this might cause dangerous conditions inside the marshes both for the local population (which gets flooded) and the vegetation (which starts dying due to the excess of water). It should also be noted that, during December 2004, the local population opened a second breach along the north embankment [2] so as to modify the amount of water entering Abu Zirig from the Islah breach.

The third location is the output from upper zone in which there are many output from this zone (between al-subahaa &al-smesm,al-subahaa and Abu Aljarry)this location represents the inlet supply water to the lower zone in the marsh.

The forth point represent the output from the lower zone and marsh these pohnts located in right and left Al-Fhud city

Table (1) presents a detailed description of each everage inlet and outlet discharge in Abu Zirig marsh for 2004 and 2005[5].

Table (1). Water balance between Upstream ,Upper &lower zone.

months	Average discharge				
	Upstream	Upper zone		Lower zone	
		In	Out	In	Out
Apr 04	46.01	32.133	18.33	18.33	1.43
May 04	33.28	22.42	14.52	14.52	12.49
June 04	34.23	28.06	18.06	18.06	16.63
July 04	25.08	24.42	16.303	16.303	15.868
Aug 04	19.28	17.5	15.214	15.214	14.703
Sep 04	32.167	28.12	16.938	16.938	15.62
Oct 04	42.067	31.774	25.882	25.882	5.877
Nov 04	24.574	18.5	9.278	9.278	6.369
Dec 04	27.45	19.867	11.25	11.25	9.375
Jan 05	42.45	33.32	21.75	21.75	17.5
Feb 05	44.56	32	20.55	20.55	17.765
Mar 05	38.65	22.42	13.45	13.45	11.45

Table (2). Monthly water quality.

Month	Sample Site	Discharge Q m^3/s	**Dissolved Oxygen** Ppm	Velociy m/s	Height m
April	AZ2	28.08	7.7	0.334	1.87
	AZ14	5.506	11.2	0.19	1.7
	AZ20	0.212	7.03	0.012	2.1
May	AZ2	16.38	8.4	0.31	1.76
	AZ14	7.989	7	0.27	1.825
	AZ20	3.745	11.2	0.57	0.65
June	AZ2	28.06	6.4	0.46	2
	AZ14	11.36	4.5	1.08	1.52
	AZ20	7.8	6.716	0.39	1.65
July	AZ2	21.6	6.64	0.28	1.56
	AZ14	9.55	3.35	0.99	1.71
	AZ20	3.637	0.1	0.24	1.25

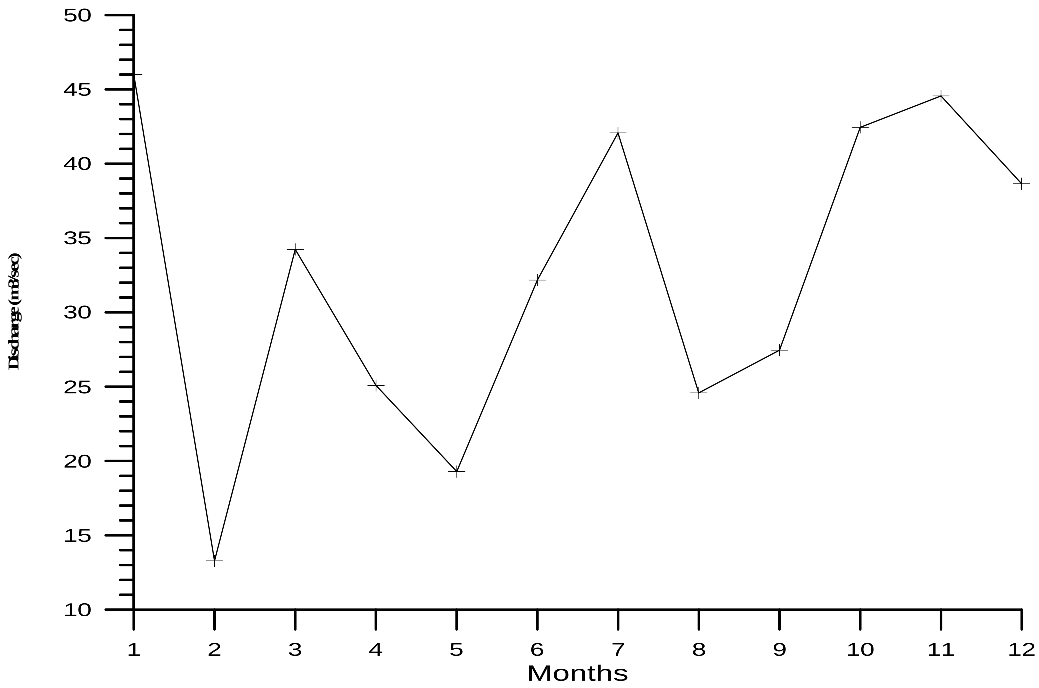

Figure (4). Discharge upstream Islah bridge in Abu Lihia river.

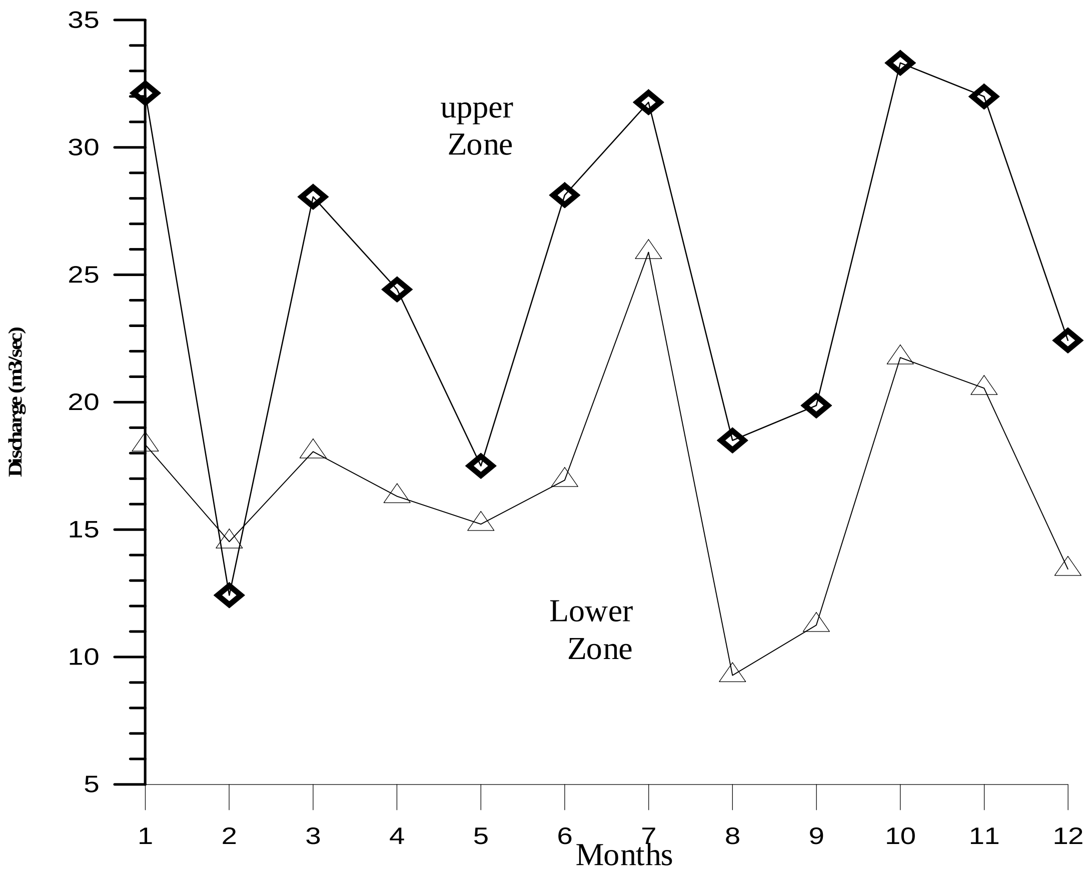

Figure (5). Inlet and outlet discharge from upper zone.

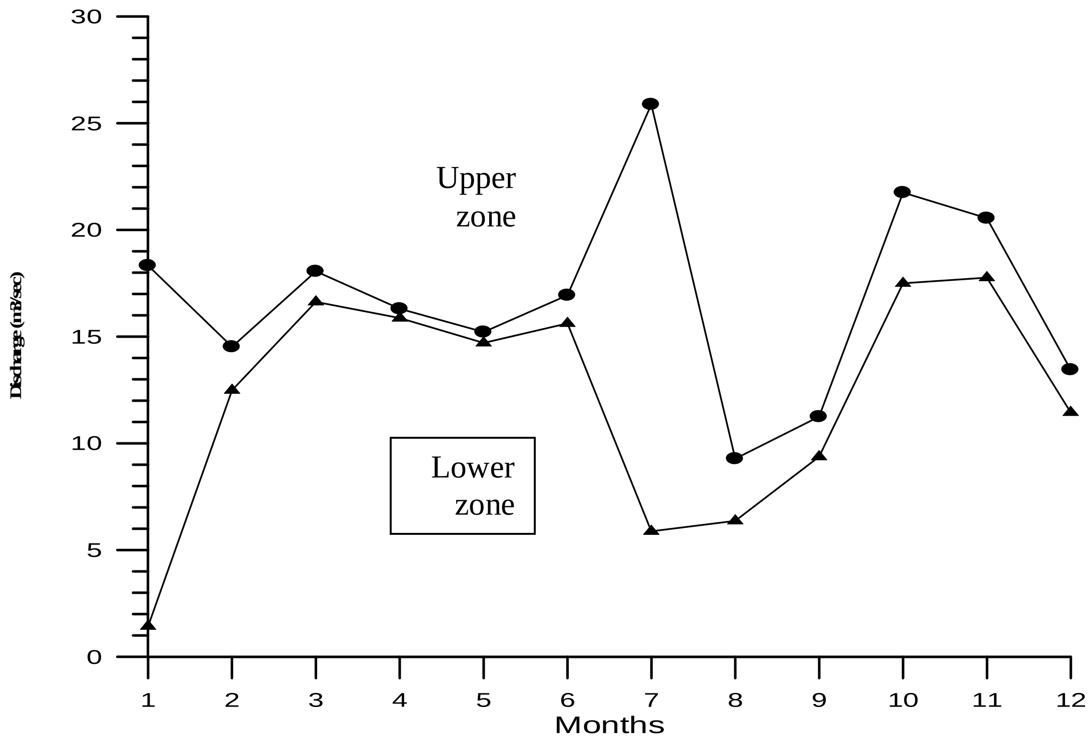

Figure (6). Inlet and outlet discharge from lower zone.

4.2 Dissolved Oxygen

Oxygen is required to support aquatic life and maintain water quality, it is the most important dissolved gas in water. Water in equilibrium with air at 25^{o}C contains 8.3 mg/L of dissolved O_2. Although water molecules contain an oxygen atom, this oxygen is not available for aquatic organisms use in natural waters. A small amount of oxygen, up to ten molecules of oxygen per million of water, is actually dissolved in water. Fish and zooplankton breath dissolved oxygen, and without sufficient oxygen mortality will occur.

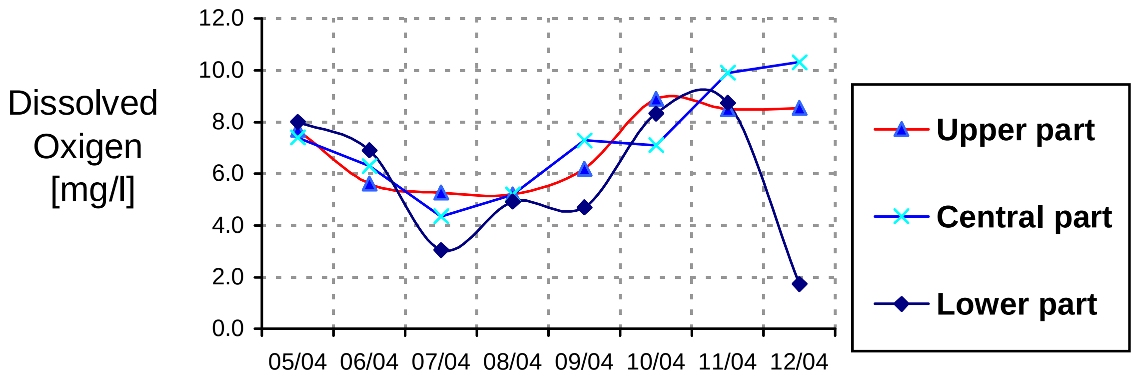

Figure (7) .Dissolved Oxygen concentration for three parts of Abu Zirig water.

5. Mass transport terms for deriving the basic model

The 1-D transport equation can be generalized for dissolved oxygen constituent as :

$$\frac{\partial M}{\partial t}=\frac{\partial\left(AD\frac{\partial C}{\partial x}\right)}{\partial x}dx-\frac{\partial(AVC)}{\partial x}dx+(Adx)\frac{dC}{dt}+S \qquad (1)$$

Where:

. M = mass (Kg)

• X = distance (m)

• t = time (Sec.)

• C = concentration (Kg m^{-3})

• Ax = cross-sectional area (m^2)

• DL = dispersion coefficient(m^2.$sec.^{-1}$)

•*U* = mean velocity (m. sec^{-1})

• S= external source or sinks (m. sec^{-1})

5.1 Assumptions and functional modifications

The basic equation solved in the study was the one dimensional advection-dispersion mass transport equation as defined by (Eq.3 and 4). In the study, this general equation was modified based on the following assumptions:

Because M = VC, we can write

$$\frac{\partial M}{\partial t}=\frac{\partial(CV^*)}{\partial t}=V^*\frac{\partial C}{\partial t}+C\frac{\partial V^*}{\partial t} \qquad (2)$$

- With steady state, non uniform flow velocity i.e $\frac{\partial Q}{\partial t}=0,\frac{\partial V^*}{\partial t}=0$.
- No external source or sink therefore, the term (S=0).
- Conservative (non-reactive contaminant) was considered $V^*\frac{dC}{dt}=0$

Divide equ.(1) by $V^*=A.dx$ becomes

$$\frac{\partial C}{\partial t}=\frac{\partial\left(D\frac{\partial C}{\partial x}\right)}{\partial x}-\frac{\partial(VC)}{\partial x} \qquad (3)$$

$$\frac{\partial C}{\partial t} = D\frac{\partial^2 C}{\partial x^2} - V\frac{\partial C}{\partial x} \quad (4)$$

Clearly, Equation (3) demonstrates the direct dependency of pollutants transport to longitudinal dispersion coefficient. Owing to the importance of longitudinal dispersion coefficient in water quality related issues, many studies have been developed to predict longitudinal dispersion coefficient in surface water after the first study of Taylor in 1953. When the 1D dispersion model is applied to predict concentration variation in surface water, the selection of a proper dispersion coefficient is the most important task. When mixing and dispersion characteristics are unknown, the dispersion coefficient can only be estimated using empirical or theoretical equations .

According to the comparative examination made on these eight methods, the equation developed by Deng et al. (2001) is found to be most reliable method for predicting longitudinal dispersion coefficient

Therefore, in the study the empirical relation developed by Deng et al (2001); Equation (5), was used for determining the dispersion coefficient of the surface water.

$$\frac{D}{HU} = \frac{0.15}{8*\varepsilon} * \left(\frac{W}{H}\right)^{5/3} * \left(\frac{V}{U}\right)^2 \quad (5)$$

$$\varepsilon = 0.145 + \left(\frac{1}{3520}\right) * \left(\frac{V}{U}\right) * \left(\frac{W}{H}\right)^{1.38} \quad (6)$$

$$U = \sqrt{gsH} \quad (7)$$

$$A = \frac{Q}{V} \quad (8)$$

$$W = \frac{V}{H} \quad (9)$$

Where:

• H = marsh depth (m)

• W = marsh width (m)

• U = marsh bed shear velocity (m/sec.)

- g= gravitational constant.
- S = slope.

5.2. Numerical solution

All the above discussions are aimed on providing the model framework for the marsh which obtains solutions numerically (or the computational representation of the marsh). Due to the complexity of water quality modeling, it has been shown that the most appropriate approach is to utilize numerical solution techniques rather than analytical approaches (Marquette, 2005).

Therefore, in this study, a forward finite difference scheme based explicit finite difference discretization method was applied.

The upper limit of the study area is located between latitude N 31º 09' 55" and latitude N 31º 06' 25". The lower limit of the river section was estimated about 10.3 Km downstream from AZ14 and located between latitude N 31º 02' 31" and latitude N 30º 59' 02".

All the above discussions were aimed to provide the model framework for the marsh. How the model obtains solutions numerically (or the computational representation of the marsh) is presented.

To apply the finite difference method, the spatial domain [0, xf was divided into M sections, each of length Δx = xf/M, and divided the time domain [0, T] into N segments, each of duration Δt = T/N, and then replace the second partial derivative on the left-hand side and the first partial derivative on the right-hand side of the equation (4) by the central difference approximation and the forward difference approximation, respectively, so that we have:

$$\frac{C_i^{k+1} - C_i^k}{\Delta t} = D\frac{C_{i+1}^k - 2C_i^k + C_{i-1}^k}{\Delta x^2} - V\frac{C_{i+1}^k - C_i^k}{\Delta x} \tag{10}$$

$$C_i^{k+1} = (1 - 2r - a)C_i^k + (r + a)C_{i+1}^k + rC_{i-1}^k \tag{11}$$

Which:

$$r = D\frac{\Delta t}{\Delta x^2} \tag{12}$$

$$a = V\frac{\Delta t}{\Delta x} \tag{13}$$

Equation (10) represent a set of simultaneous linear equations whose solution provides the values of for all **i**'s. MATLAB computer language was used for solving this problem.

The model input data has an average values for flow rates, velocity, slope, marsh depth and width.

Initial concentration of dissolved Oxygen:

C(x,0) = 0 for lower and upper parts if the marsh.

Boundary conditions with average values:

C(0,t) = 7.3 (upper part) and 6.6 (lower part).

C(xf,t) = 6.6 (upper part) and 6.25 (lower part)

6. Result and discussion

The contaminant transport model was solved by implementing the explicit finite difference numerical scheme, while varying the different parameter values. In general, a decrease in contaminant concentration was observed. The concentration of the point source measured for different time periods. The results calculated using the model are displayed in Figures (8,9,10 and 11).As shown in the figures the peak of the spill spreads as time progresses and progressively decays due to dispersion. As it was shown in Figure (8), the peak point reaches about 2 mg/l at t= 10 min.(r=0.1) for upper part and less than 2 mg/l for the lower part In Figure (10) and (11), after 1/2 hr time (r=0.3), the same effect but more decay in concentration due to the dispersion of the chemical and this peak point moved to the last nodal point due to advection mass transport.

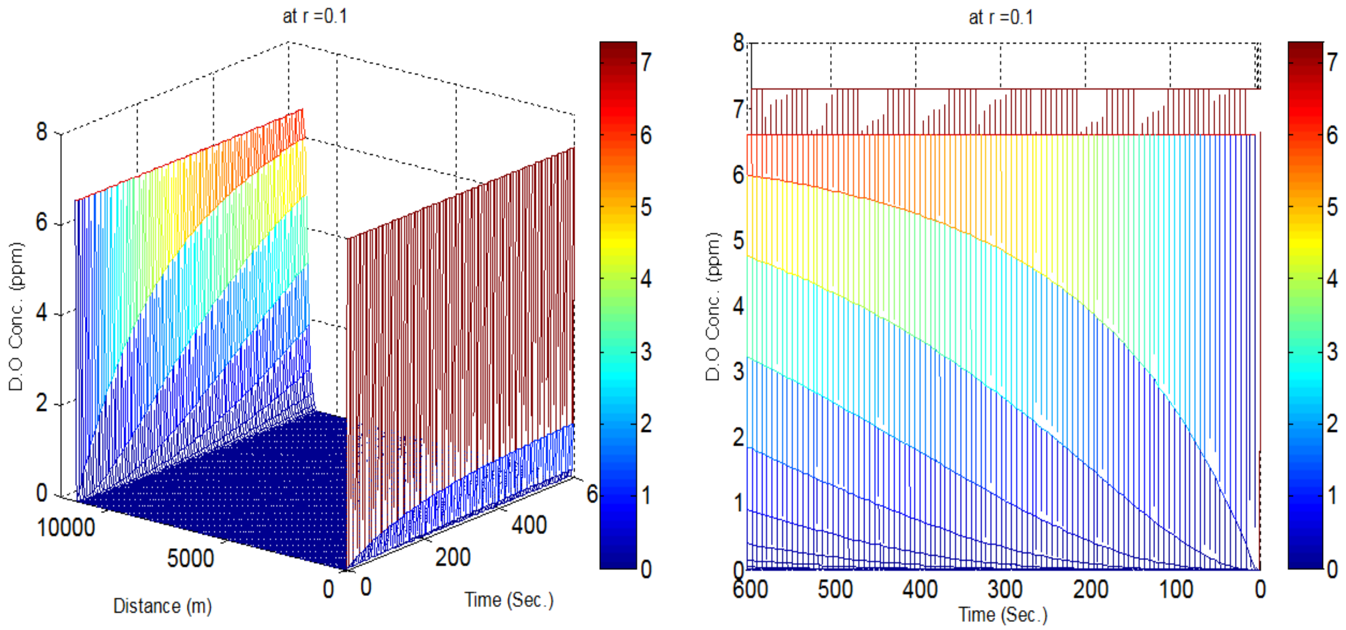

Figure (8).The spatial distribution of D.O conc. in the upper part after 10min .

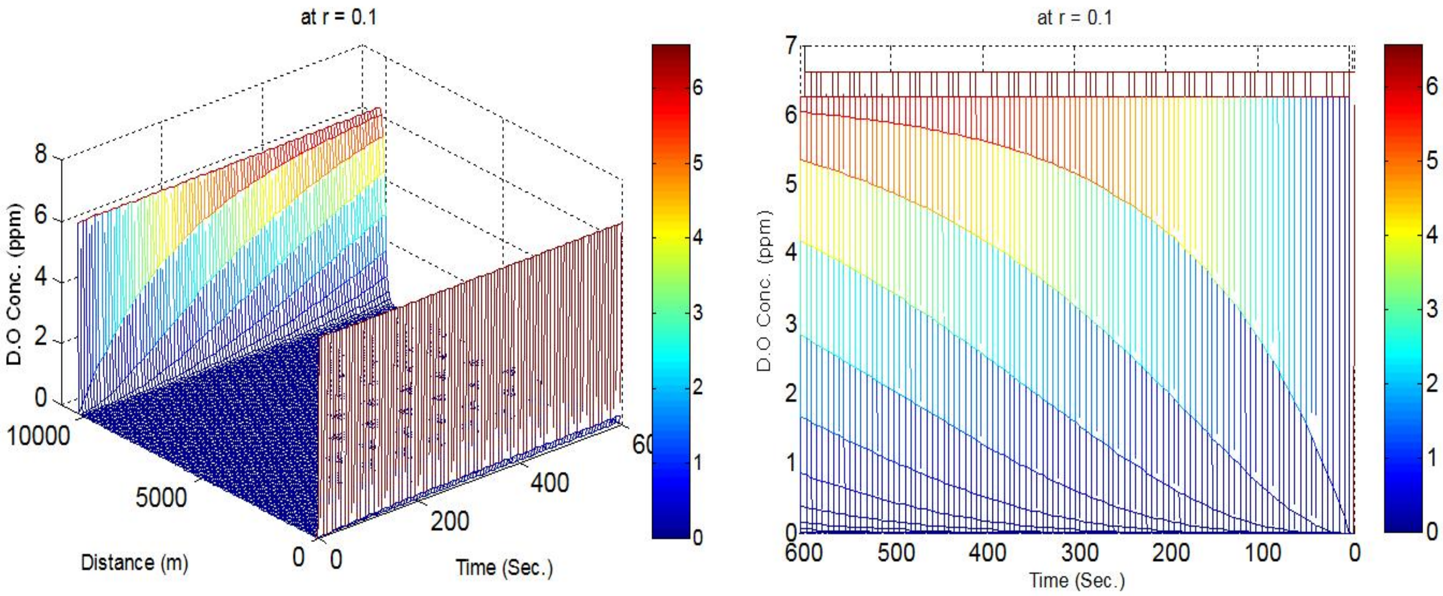

Figure (9).The spatial distribution of D.O conc. in the lower part after 10min .

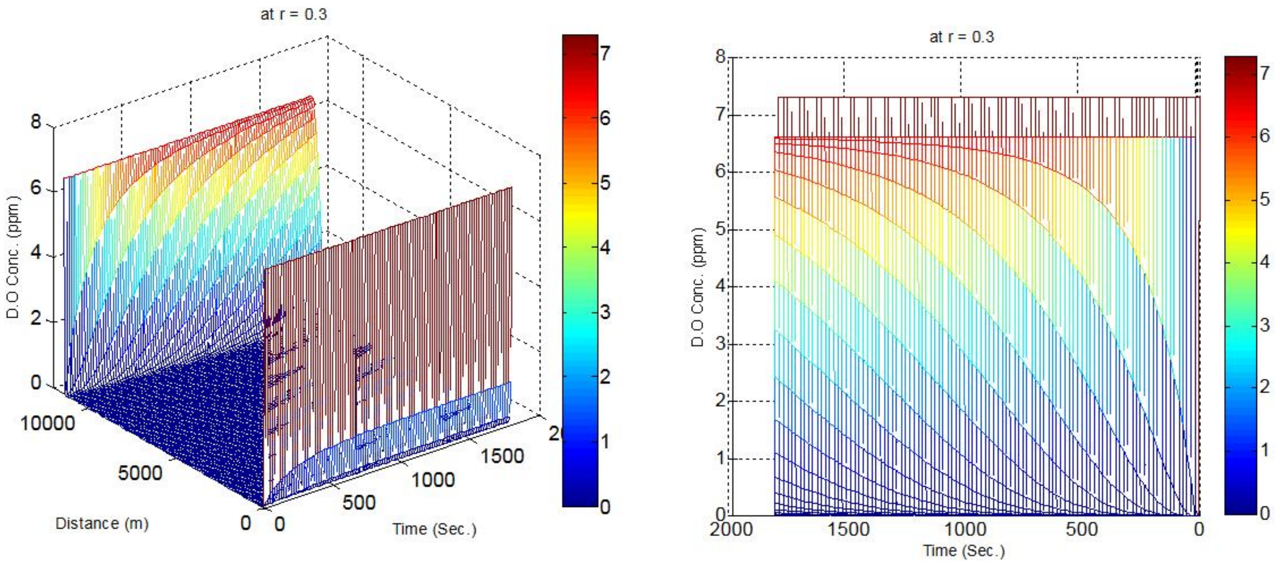

Figure (10).The spatial distribution of D.O conc. in the upper part after 1/2 hr .

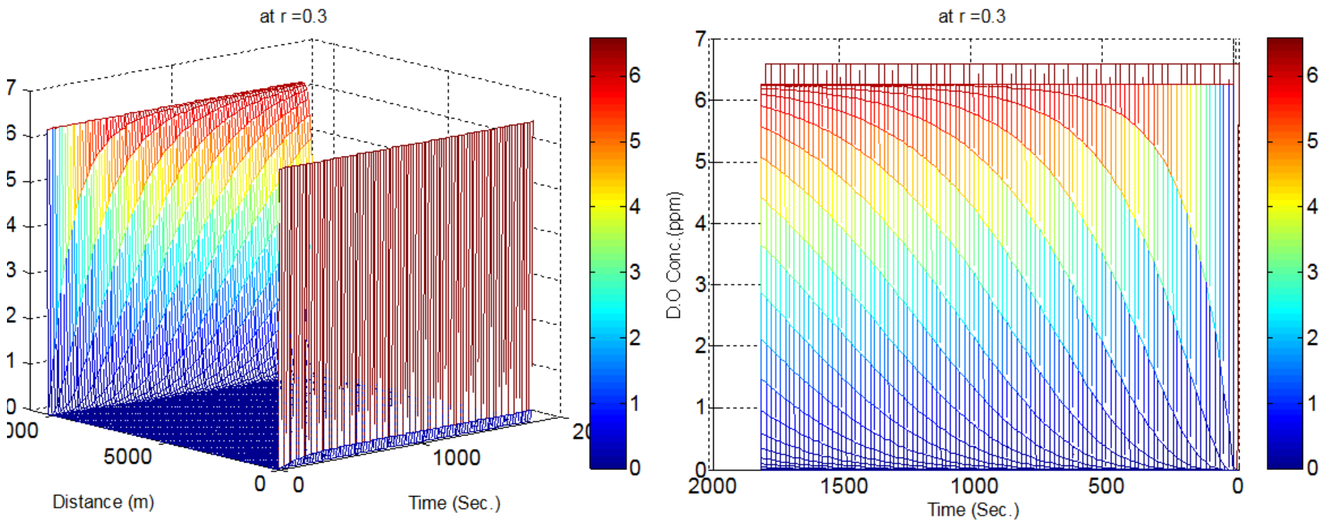

Figure (11).The spatial distribution of D.O conc. In the lower part after 1/2 hr .

As it was shown in all Figures, the concentration of the conservative chemical reaches a constant concentration level. A transition approaches a constant pattern, which is an asymptotic shape in which a zero concentration gradient reached with the upstream. The time required for reaching this onstant concentration point varies from location to location. Such curves are referred to as “breakthrough” curves and they are used extensively in both surface water problems context. One of the important uses of these breakthrough curves is in determining the rivers flow and transport properties, particularly for determining the mean advective flow velocity and longitudinal dispersion coefficient of marsh and surface water. As displayed .

7. Conclusion

This paper describes the development of explicit finite difference central scheme for an advection diffusion method. The final model was\used to predict the dissolved oxygen concentration. A general one – dimensional computer based mechanistic water quality model has been developed for the upper and lower sections of the Abu-Zirig marsh water. This general water quality model can represent the fate and transport of any contaminant transport in the long sections of the Marsh system.

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