# Experimental Study of Fluid Flow Through Packed Beds of Glass Sphere Packing 

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#### Abstract

Fluid flow through packed bed has many important applications in chemical and other process engineering fields such as fixed-catalytic reactor, adsorption of a solute, gas absorption, combustion, drying, filter bed, wastewater treatment and the flow of crude oil in petroleum reservoir.

The present work presents the study of air flow through a packed bed of glass sphere packing with $0.42,0.50,0.61,0.79$ and 1.01 cm in diameter, and the packed column was 7.62 cm in diameter and 57 cm long. Different flow rates of fluid were used which expressed by modified Reynolds number. Many variables were studied in this work such as fluid type (air flow), flow rate and the packing porosity, in order to study the effect of these variables on the friction factor.

Anew correlation for friction factor as a function of Reynolds number for air flow through packed of mono size packing has been made.

The results showed that the pressure drop through a packed bed is highly sensitive to the packing porosity which has a significant effect on the friction factor. It was found that as the bed porosity increases the friction factor values as well as the pressure drop values decrease.


Key Words: Friction Factor, Packed Bed, Reynolds Number, Porosity, Air Flow.

# الدراسة العملية لجريان الموائع خلال العمود الحشوي المحشو بجسيمات زجاجية كروية الثكل 

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جريان الموائع خلال الاعمدة المحشوة لها تطبيقات مهـة في الهندسة الكيمياويـة و الفروع الهندسية الاخرى، مثل
المفاعلات المحفزة والامتزاز من المذاب ، امتصـاص الغـاز ، والاحتراق ، و التجفيف ، وتصفية المسرير ، ومعالجـة مياه
    الصرف الصحي وتدفق النفط الخام في المكامن النفطية.
تم في هذا البحث در اسة جريان الهواء خلال عمود حشوي محشو بجسيمات زجاجية كروية الثكل وبأ قطـار 0.42,
        0.5, 0.61، 0.79 و 1.01 سم، و كان قطر العمود الكحشو 7.62 سم و طولـه 57
    الجريان الموائع و الممثلة بعدد رينولاز . لقد تم دراسة عدة عوامل خـلال هذا العمل كنوع المـانع (جريـان الهواء),
        الجريان وتغير مسامية الحشوة لمعرفة تاثير هذه العو امل على معامل الاحتكاك.
        تم ايجاد علاقة تجرييية جديدة تربط بين معامل الاحتكاك ورقم رينولاز لجريان الهواء خـلال
                                    على تجاربنا العطلية.
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اثبتت نتائج الدر اسة ان هبوط الضغط خلال عمود حشوي ينأثر للدجة كبيرة بمسامية الحشوة والتي لها تأثنير ملموس . حيث لوحظ انـه عندما تز داد مسـامية الحشوة تقل قيم معامل الاحتكـاك, قيمـة هبوط

## 1- Introduction

The study of fluid flow through the packed bed is an important issue. Chemical engineering operations commonly involve the use of packed and fluidized beds. These are devices in which a large surface area for contact between a liquid and gas (absorption, distillation) or a solid and a gas or liquid (adsorption, catalysis) is obtained for achieving rapid [1].

A packed bed is simply a vertical column partially filled with small media varying in shape, size, and density. A fluid (usually air or water) is passed thought this column from the bottom and the pressure is measured by two sensors above and below the packed bed [2]. Packed beds are consists of a channel or duct which contains some form of porous material or a collection of randomly packed spheres or other non-spherical particle [3]. The packing material may be glass marbles, ceramics, plastics, pea gravel, or mixtures of materials [4]. It should have a large void volume to allow flow of fluid without excessive pressure drop and it should be chemically inert to fluids being processed [5]. The advantage of using packed column rather than just tank or other reaction vessel is that the packing affords a large contacting surface area for fluids to flow. Usually increased surface area provides a high degree of turbulence in the fluids which are achieved at the expense of increased capital cost
and/or pressure drop, and a balance must be made between these factors when arriving at an economic design [6]. The fluid path is made of many parallel and interconnecting channels. The channels are not of fixed diameter but widen and narrow repeatedly, and even twist and turn in varying directions as the particles obstruct the passageway [7]. Owing to the complicated nature of the flow channels in granular bed, there should not be a sharp transition from laminar to turbulent flow as occurs in pipe flow. Rather there should be a smooth transition from laminar flow throughout - to laminar flow in parts of the granular bed and turbulent flow in other parts - to turbulent flow throughout [8]. The flow rate of fluid is important factor affecting on the pressure drop through packed bed. When there is no flow through the packed bed, the net gravitational force acts downward. When fluid flows upwards, friction forces act upward and counter balance the net gravitational force [9, 10]. From the reading of the manometers, Coulson in 1949 found that the difference in pressure over varying thicknesses of the packing was obtained directly. Some results for bed of spherical particles are shown graphically in Fig 1. The experimental points are seen to lie on straight lines indicating a linear relation between $\Delta \mathrm{p}$ and L. Figure 1 also shows similar results for beds of other material [11].

The fluid flow through packed bed has attracted considerable attention from many investigators; they have shown that the most important issue for mechanical perspective for liquid or gas flow through packed bed depends on the pressure drop and friction [2]. The first carefully documented friction experiments have been carried out by Hagen in 1839 for laminar flow [12]. Schoenborn and Dougherty in 1944 studied the flow of air, water and oil through beds of various commercial ring and saddle packing [13]. Harkonen in 1987, Lindqvist in 1994, Lammi in 1996, Wang and Gullichsen in 1999 and Lee and Bennington in 2004 measured the average void fraction and flow resistance through packed columns. They found that the pressure drop of liquid through a packed bed depends on many factors, including the particle species and the type and size distribution of the particles. An attempt has been made by $\mathbf{Y u}$ and Standish in 1989 to establish general theory of the random packing of particles. They developed an analytical model based on the experimental results of binary mixtures [14]. Basu et. al. in 2003 studied the effect of various velocity range on the packed bed column and took their observations of the packing height and pressure drop in the column [15].

The packing of solid particles has been studied more or less continuously for a number of years. The first study of the modes of packing of spheres appears to have been undertaken by Sticker in 1899 [8]. Furnas in 1931 [15], Westman 1930 [17], studied the packing of a bed
of different sizes solid particles [10]. Graton in 1985 [17] studied the packing of spheres led to the much-quoted limits of porosity for regular packing of single-size spheres.


Figure (1). Relation between depth of bed and pressure drop.

Many studies have been done in correlating data for packed columns at higher fluid velocities. Blanke in 1926 suggested that this change of relationship between pressure drop and velocity is entirely analogous to that which occurs in ordinary pipes and proposed a friction factor plot similar to that of Stanton [18]. The equation used was that for kinetic effect modified by a friction factor, which is turn, is a function of Reynolds [19].

$$
\begin{align*}
& \Delta p=2 f\left(\rho u^{2} / D_{p}\right)  \tag{1}\\
& f=\phi(N \mathrm{Re}) \tag{2}
\end{align*}
$$

Some workers have included the effect of void fraction by the addition of another factor in equ. 2. It is usually given in the form $(1-\varepsilon)^{\mathrm{m}} / \varepsilon^{3}$ where m is either 1 or 2 .

Carman in 1938 and Kozeny in 1927 suggested that the change of relationship between pressure drop, void fraction effect and velocity, and proposed a friction factor for entire Reynolds number by plotting on a logarithmic basis [20]. Carman correlates his data of the friction factor as a function of the Reynolds number for condition of fixed bed operation; the main variables are the velocity, particle diameter, pressure drop per unit length and fraction voids. Carman correlation for fluid flow through randomly packed beds of solid particles by a single curve whose general equation was:

$$
\begin{equation*}
f=5 \mathrm{Re}_{1}^{-1}+0.4 \mathrm{Re}_{1}^{-0.1} \tag{3}
\end{equation*}
$$

Where $\mathrm{Re}_{1}$ is the modified Reynolds number and can be expressed in the following equation:

$$
\begin{equation*}
\operatorname{Re}_{1}=\frac{\rho u}{S(1-e) \mu} \tag{4}
\end{equation*}
$$

And $S$ is the specific surface area of the particles and is the surface area of particle divided by its volume. Its units are (length) ${ }^{-1}$. For sphere:

$$
\begin{equation*}
S=\frac{\pi d_{p}{ }^{2}}{\pi\left(d_{p}{ }^{3} / 6\right)}=\frac{6}{d_{p}} \tag{5}
\end{equation*}
$$

The general surface of a bed of particles can often be characterized by the specific area of the bed $\left(S_{B}\right)$ and the fractional voidage of the bed $(\varepsilon)$. $S_{B}$ is the surface area presented to the fluid per unit volume of bed when the particles are packed in bed. Its units are (length) ${ }^{-1}$. It can be seen that $S$ and $S_{B}$ are not equal due to the voidage occurring when the particles are packed in to a bed. If contact points occur between particles so that only a very small fraction of surface area is lost by overlapping, then [8]:

$$
\begin{equation*}
S_{B}=S(1-\varepsilon) \tag{6}
\end{equation*}
$$

For a given shape of particle S increases as the particle size is reduced. When mixtures of sizes are studied the value of $S$ for sphere of mixed sizes is given by [21]:

$$
\begin{equation*}
S=6(1-\varepsilon) \sum \frac{x_{i}}{d_{p i}} \tag{7}
\end{equation*}
$$

Where: $x_{i}$ is the fractional weight of spherical particle; $d_{p}$ is the diameter of spherical particle.

Sawistowski in 1957 has compared the results obtained for flow of fluids through beds of hollow packing and has noted that equation 3 gives lower values of friction factor for hollow packing [22]. Thus, Sawistowski modified equation 3 as:

$$
\begin{equation*}
f=5 \mathrm{Re}_{1}^{-1}+\mathrm{Re}_{1}^{-0.1} \tag{8}
\end{equation*}
$$

Ergun in 1952 studied the pressure drop and friction factor through packed beds composed of uniform spherical particles [9]. His model was also used for non-spherical shape
and/or the particle size distribution was non-uniform [2]. His equation can provide the pressure drop along the length of the packed bed given a fluid velocity:

$$
\begin{equation*}
f=4.17 \mathrm{Re}_{1}^{-1}+0.29 \tag{9}
\end{equation*}
$$

Many attempts to study the effect of surface roughness on the friction factor have shown that the variable has a significant effect but no quantitative method of evaluating the effect has been formulated. Leva in 1949 carried out important experiments using three different groups of materials. He used glass and porcelain as smooth particles, clay and Alundum as rough materials, Aloxite and MgO granules as rougher materials. The degree of the materials roughness was described qualitatively (as a result of his tests). This method of description does not help in the development of a quantitative correlation between the relative roughness $(\varepsilon / \mathrm{d})$ and the friction factor. Leva expressed equations for the friction factor of these materials as follow. For smooth materials the expression was:

$$
\begin{equation*}
f=\frac{1.75}{\mathrm{Re}_{1}{ }^{0.1}} \tag{10}
\end{equation*}
$$

For rough materials the expression was:

$$
\begin{equation*}
f=\frac{2.625}{\operatorname{Re}_{1}^{0.1}} \tag{11}
\end{equation*}
$$

Whilst for rougher materials the expression was:

$$
\begin{equation*}
f=\frac{4.0}{\operatorname{Re}_{1}^{0.1}} \tag{12}
\end{equation*}
$$



Figure (2). Friction factor for various materials [23].
Figure (2) shows that the friction factor through packed beds is approximately doubled when the degree of surface roughness was increased from that represented by porcelain to the roughness of Aloxite granules [23]. These expressions do not include the relative roughness $(\varepsilon / d)$ and can only therefore be used for the materials that were used. Leva concluded (from previous expressions) that the degree of surface roughness had no effect on the slope of the $\log f-\log \operatorname{Re}$ curve between the limits of Re from 130 to 7000 which covered the range was used [24].

Aim of the work:
The aim of this work is to propose an empirical correlation between friction factor and Reynolds number for air flow through bed of spheres, and study the effect of particle size distribution on the bed porosity. Also studying the effect of bed porosity on the pressure drop and friction factor through the packed bed.

## 2- Experimental work

In this work five sizes of spherical glass particles were used. The spherical particles diameters were $0.42,0.51, .61,0.79$ and 1.01 cm . The fluid used was air, the properties was
taken at city temperature $\left(31^{\circ} \mathrm{C}\right)$. The physical properties of air at this temperature are density $1.1582 \mathrm{~kg} / \mathrm{m}^{3}$ and viscosity $1.88 * 10^{-5} \mathrm{~kg} / \mathrm{m} . \mathrm{s}$.

A schematic diagram of the apparatus used is shown in Figure3. The packed bed column was made of Pyrex glass tube (Q.V.F) 7.62 cm inside diameter and 57 cm height. The Q.V.F glass contains two taps at the inlet and outlet of the column. The taps used for measuring the pressure drop are placed flush to the inside surface to determine the static pressure actuarially. The first one was placed downstream at a distance 1 cm from the sieve entrance regional and the second tap was placed at distance 1 cm from the top sieve of the column. The distance between the inlet and outlet to the column and the sieve was to avoid the turbulence at the bed. The column was mounted vertically and holds by iron flanges.

Air flow is produced by a compressor to the packed column. The compressor contained a vane rotary type driven by AC motor. The compressor was used to supply the air to packed bed at constant pressure. A storage tank was used to receive air from the compressor and provide it to the rotameter. A rotameter was used for measuring air flow rate, and the flow rate up to 16 cubic meters per hour. The U-tube manometer (with ethanol) was used for measuring the pressure drop through packed column.


Figure (3). Air flow through packed bed.

The particles were poured into the column until it was filled and the bed porosity was determined using the following equation:

$$
\begin{equation*}
\varepsilon=1-\frac{\rho_{b}}{\rho_{t}} \tag{13}
\end{equation*}
$$

Where: $\rho_{\mathrm{b}}$ is the bulk density $\left(\mathrm{g} / \mathrm{cm}^{3}\right), \rho_{\mathrm{t}}$ is the true density of particles $\left(\mathrm{g} / \mathrm{cm}^{3}\right)$.

The air was provided by the compressor and its flow rates up to 16 cubic meters per hour. It was controlled by means of a control valve at the inlet of the rotameter. The average velocity of the air was obtained from rotameter using equation 14.

$$
\begin{equation*}
u=\frac{Q}{A} \tag{14}
\end{equation*}
$$

Where Q is the flow rate of fluid $\left(\mathrm{m}^{3} / \mathrm{hr}\right)$, A is the bed cross-sectional area $\left(\mathrm{m}^{2}\right)$.

The rotameter valve was opened for air flow. The rotameter float was ensured to achieve steady state, and the pressure drop across the bed was measured using u-tube manometer. The friction factor was obtained from pressure drop using the following equation [5]:

$$
\begin{equation*}
\frac{R_{\mathrm{i}}}{\rho u_{1}^{2}}=\frac{\varepsilon^{3}(-\Delta p)}{S(1-\varepsilon) L \rho u^{2}} \tag{15}
\end{equation*}
$$

Bulk density is defined by the following expression:
bulk dencity $\left(\rho_{\mathrm{b}}\right)=\frac{\text { weigth of the bed }}{\text { volume of bed }}$
For a cylindrical bed

$$
\begin{equation*}
\text { volume }=\frac{\pi}{4} D^{2} L \tag{17}
\end{equation*}
$$

Where: $\mathrm{D}=$ inside diameter of the cylinder $(\mathrm{cm}), \mathrm{L}=$ level of the particles in the bed $(\mathrm{cm})$.

The true densities of particles were determined using shifted water method. To obtain the volume of samples a known weight of particles was immersed in a graduated cylinder (with capacity of 500 ml ) filled with water. The weight of container was measured using a sensitive balance first when the container filled with water only and second when it contains the particle besides the water. In both cases, water level inside the container was carefully
maintained at it permissible full mark level. Using the following equation, true density of particles was being determined.

$$
\begin{equation*}
\rho_{t}=\frac{w_{1} \times \rho_{w}}{w_{2}-w_{3}+w_{1}} \tag{18}
\end{equation*}
$$

Where: $\rho_{\mathrm{w}}=$ water density at laboratory temperature $\left(\mathrm{g} / \mathrm{cm}^{3}\right), \mathrm{w}_{1}=$ solid particles weight $(\mathrm{g})$, $\mathrm{w}_{2}=$ weight of cylinder filled with water $(\mathrm{g}), \mathrm{w}_{3}=$ weight of cylinder with water and particles (g).

For mixture of particles, the mixture true density $\left(\rho_{t m}\right)$ can be determined from the following equation:

$$
\begin{equation*}
\rho_{t m}=\frac{1}{\sum_{i=1}^{m} \frac{x_{i}}{\rho_{t i}}} \tag{19}
\end{equation*}
$$

Where $x_{i}$ is the weight percent of component $i, \rho_{t i}$ is the true density of component $I\left(g / \mathrm{cm}^{3}\right)$.

## 3- Result and discussion

The values of friction factors for air flow through beds of mono sizes particles are plotted versus Reynolds numbers as shown in Figures (4-8). These Figures show that the friction factor decreases as Reynolds number increases.

The wall affects the bed porosity and increases its value. This appears in Figure (8) where the bed porosity increases to a value of 0.4359 , this wall effect may be due to the ratio of bed diameter $(7.62 \mathrm{~cm})$ to the particles diameter $(1.01 \mathrm{~cm})$ which is less than the supposed ratio (column diameter to the particle diameter should be greater than 10:1).

Figures $(4-8)$ show that the values of friction factor of Figure (4) decrease sharply with increasing Reynolds numbers while that of Figure (8) decrease slightly with increasing Reynolds number, because the fluid flow of Figure (4) is at the laminar and transition regions (where the friction factor-Reynolds number curve is of slope of -1 ) while the fluid flow of Figure (8) is at the transition and turbulent regions (where the friction factor-Reynolds number curve become straighter).


Figure (4). Friction factor versus Reynolds numbers for particles diameter of 0.42 cm and porosity of 0.3746 .


Figure (5). Friction factor versus Reynolds numbers for particles diameter of $\mathbf{0 . 5 1} \mathbf{~ c m}$ and porosity of $\mathbf{0 . 3 9 9 9}$.


Figure (6). Friction factor versus Reynolds numbers for particles diameter of $\mathbf{0 . 6 1} \mathbf{~ c m}$ and porosity of $\mathbf{0 . 4 1 1 2}$.


Figure (7). Friction factor versus Reynolds numbers for particles diameter of $\mathbf{0 . 7 9} \mathbf{~ c m}$ and porosity of $\mathbf{0 . 4 2 2 5}$


Figure (8). Friction factor versus Reynolds numbers for particles diameter of $1.01 \mathbf{c m}$ and porosity of $\mathbf{0 . 4 3 5 9}$

The difference between the experimental results and the results of Ergun, Carman and Sawistowski, this is may be due to the fact that Ergun and Sawistowski used hollow packings which have certain values of sphericity and porosity (the lower the particle sphericity the more open is the bed) while Carman used sphere packing [9, 20, 22].

It is clear that as the porosity decreases the friction factor decreases [25], in spite of pressure drop increases, and this is because that the friction factor is proportional to power three with porosity, while it is proportional to power one with pressure drop as shown in the equation below:

$$
f=\frac{\Delta \mathrm{P}}{L} \frac{d_{P}}{\rho u^{2}}\left(\frac{\varepsilon^{3}}{1-\varepsilon}\right)
$$

The best fitting of the experimental results for air flow through beds of mono-sizes particles is represented by the following equation.

$$
f=4.4296 \operatorname{Re}^{-1}+0.5489 R e^{-0.1}
$$

With the correlation coefficient is 0.96and percentage of average errors is $3.446 \%$.

## 4- Conclusion

The experimental results show that the friction factor decreases as Reynolds number increases. Examining the experimental results it can be seen that the pressure drop in the bed is inversely proportional to bed porosity for the same velocity of the fluid entering the bed.

The curves representing the experimental results of friction factor for air flow indicate that the air flow was intermediate between the turbulent and laminar regions (at the turbulent region the Reynolds number have insignificant effect on the friction factor values).

The bed porosity highly affects the pressure drop and inversely proportional to it, this is because that when the porosity increases the resistance to fluid flow through the bed decreases.

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## 6- Notations

## Symbols

## Notations

$K^{\prime \prime}=$ Kozeny constant
$\Delta \mathrm{p}=$ Pressure drop through packed bed (kg/m.s. ${ }^{2}$ )
$\mathrm{u}=$ Superficial velocity $(\mathrm{m} / \mathrm{s})$
$\mathrm{L} \quad=\quad$ The height of packing in the bed $(\mathrm{m})$
$\mathrm{Q}=$ Flow rate $\left(\mathrm{m}^{3} / \mathrm{hr}\right)$
$\mathrm{S}=$ Specific surface area of the particles $\left(\mathrm{m}^{2} / \mathrm{m}^{3}\right)$
$S_{B}=$ Specific surface area of the bed $\left(\mathrm{m}^{2} / \mathrm{m}^{3}\right)$
$\mathrm{A}=$ The bed cross-sectional area $\left(\mathrm{m}^{2}\right)$
$\mathrm{d}_{\mathrm{p}} \quad=\quad$ Diameter of the particle $(\mathrm{m})$
$\mathrm{d}_{\text {pav }} \quad=\quad$ Average particles size (m)
$\mathrm{d}_{\mathrm{pi}}=$ Diameter of particle i in mixture (m)
$\mathrm{D}=$ Diameter of cylinder (m)
$\mathrm{Re}_{1}=$ Modified Reynolds number
$\mathrm{e}=$ Porosity of the bed
$\frac{R_{1}}{\rho u_{1}^{2}}=$ Modified friction factor
$\mathrm{u}_{1}=$ Average velocity through the pore channels $(\mathrm{m} / \mathrm{s})$.
$L^{\prime} \quad=\quad$ Length of channel (m)
$\mathrm{d}_{\mathrm{m}} \quad=\quad$ Equivalent diameter of the pore channels (m)
$K^{\prime} \quad=\quad$ Is a dimensionless constant whose value depends on the structure of the bed
$X_{i}=$ The proportion of the component i in the mixture
$x_{i}=$ The weight fraction of particle i
$f_{w}=$ Correction factor
$S_{c} \quad=\quad$ Surface of the container per unit volume of bed $\left(\mathrm{m}^{-1}\right)$
$\mathrm{d}_{\mathrm{t}} \quad=\quad$ Diameter of tube $(\mathrm{m})$
$f=$ Modified friction factor
$\mathrm{q}=$ Number of components in the mixture
a $\quad=$ Representation of packing and fluid characteristics at laminar flow
$\mathrm{b}=$ Representation of packing and fluid characteristics at turbulent flow

## Greek Symbols

$\varepsilon \quad=$ Porosity of the bed
$\rho_{b} \quad=$ Bulk density $\left(\mathrm{g} / \mathrm{cm}^{3}\right)$
$\rho_{t} \quad=$ True density $\left(\mathrm{g} / \mathrm{cm}^{3}\right)$
$\rho_{t m}=$ True density of mixture $\left(\mathrm{g} / \mathrm{cm}^{3}\right)$
$\rho_{t i}=$ True density of component i
$\rho=$ Density of fluid $\left(\mathrm{kg} / \mathrm{m}^{3}\right)$
$=$ Fluid viscosity (kg/m.s)
$\emptyset_{s}=$ Sphericity
$\delta=$ orientation factor
$\theta=$ angle of the solid liquid interface with the stream direction

