

Effect of an Impervious Core Constructed into a Large Earth Dam on the Quantity of Seepage

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Abstract

Impervious core is used in zoned earth dam to reduce the quantity of seepage through the body of the dam and to relief part of risk of piping and erosion in the downstream side. This paper presents cases study of the effect of core permeability, thickness and location on the flow through large earth dam. Finite element method was used to solve the problem of flow of incompressible fluid which is governed by Laplace equation. A considerable range of shell permeability to core permeability was used. The study has shown that quantity of seepage was reduced as ratio of shell to core permeability increases. The reduction in seepage continues to a limited value of core permeability after which the effect decreases. Doubling the core base thickness reduces the quantity of seepage highly and inclination of core towards upstream side slightly increases the quantity of seepage.

Keywords: Earth dam; Finite element; free surface; core; seepage.

تأثير لب مانع للتسرب منشأ في سد ترابي كبير على كمية النضوح

المستخلص

يستعمل اللب المانع من التسرب في السدود الترابية لتقليل كمية التسرب خلال جسم السد و لإزالة جزء من مشكلة التآكل في الجهة الثانية من السد. يقدم هذا المقال دراسة حالات تأثير نفاذية اللب وسمكه وموقعه على النضوح خلال سد ترابي كبير. تم استخدام طريقة العناصر المحددة لحل مسألة نضوح مائع غير قابل للانضغاط والذي يحكم بمعادلة لابلاس. تم استخدام مدى واسع من نفاذية جوانب السد إلى قيم نفاذية اللب. أوضحت الدراسة تناقص كمية التسرب كلما ازدادت نسبة نفاذية جوانب السد إلى نفاذية اللب. يستمر التناقص في التسرب إلى حد معين من نفاذية اللب بعدها يكون التأثير قليلاً. وجد بان مضاعفة سمك اللب يقلل كمية النضوح بشكل كبير كما وجد بان انحراف اللب باتجاه جهة مقدم السد يزيد وبشكل قليل كمية الجريان.

1. Introduction

Excessive seepage forces may cause instability of soil mass in earth dam at the exit or exposed face in a form of piping and heave if they exceed the accepted limit. To eliminate this risk three process should be followed through design stage to control the seepage: preventing piping and heave, reducing seepage and draining the water [1]. Reducing the seepage through dam section may be achieved by using impervious membrane on the upstream or impervious cores. Design of impervious cores may consider several parameters such as thickness, slope, location of core whether it is installed vertically or installed towards upstream side and the permeability of construction materials.

A main requirement of core is that it should provide the necessary degree of imperviousness, and should be deformable to withstand differential settlement without cracking. The imperviousness, hydraulic gradient and cracking may be influenced by thickness of core, for example thin cores may associate with high seepage gradient while thick core may associate with low hydraulic gradient [2]. Although the effect of core thickness on some elements of earth dams has been studied by some researchers [3, 4,5, 6], lack of information about the effect of thickness on seepage through earth dam such as free surface, surface seepage and quantity of seepage still keep on.

Free surface was calculated by two methods fixed mesh [7,8] and adaptive mesh [9,10]. In the adaptive mesh algorithm, the free surface requires implementation of Darcy's law by finite element and procedure to add-on mesh deformation. Adaptive mesh is simpler more than the fixed mesh [11]. Indicating the location of free surface has a great attention in determining the exit point of flow at the downstream side. Therefore it may be affected by location of the core whether it is constructed vertically or inclined towards upstream side. The researches on this aspect are few and studying the effect of location of core on the quantity of seepage will increase the information on controlling the seepage through the dam.

This research adopts a finite element model to solve the unconfined flow through large earth dam. After verifying the model with available solution for a typical problem, a core with different thickness and location are investigated. The main object of this research is to study the flow through earth dam with different core permeability, core thickness and core location by using finite element method with adaptive mesh method. It also objects to (1) quantify the quantity of seepage and located the free surface; (2) investigate two core locations and (3) study the effect of core permeability on the quantity of seepage.

2. Finite element model

Unconfined flow through earth dam is assumed to be governed by Darcy's law in which the velocity correlated to head between any two points in the flow media

$$v = -k \text{ grad}(h) \quad (1)$$

where v is the velocity, k is the permeability, h is the head and pressure. Using conservation of water leads to the following partial differential equation assuming steady state flow

$$\text{div}(v) = 0.0 \quad (2)$$

$$k \nabla^2 h = 0.0 \quad (3)$$

The partial differential equation is available in common procedures produced by Smith and Griffiths [10]. The degree of freedom is the pressure which can be implemented through programming the finite element to compute the quantity of seepage and to locate the free surface.

3. Verification the model

The computer program of flow in two dimension (program 7.3) [10] based on finite element method outputs the pressure value, inflow and outflow at every point in the mesh and also locates the free surface. The characteristics of the problem solved herein as an example for verification purposes are a flow through a rectangular dam with 10m height, 5m width, 10m upstream head and 2m downstream head. The media of flow was divided into 50 elements: 10 elements in y direction and 5 elements in x direction. Figure shows the mesh of the verified problem that uses a rectangular quadrilateral element with four nodes.

The computer program results are plotted in a mesh form in which every node represents the pressure head Figure (2). Free surface was represented by a solid line and the numerical solution results obtained by other researchers were also presented. The free surface starts from the top point at the right hand side and ends at a point at extreme left side where head pressure approximately equals 6.5 m. This result approximately agrees with results found in the previous researches [12,8, 13]. In spite of low tail- water height (2 m) the elevation of the exit point of the free surface was 6.5 m. This value is approximately the same value (6.2 m) obtained by Polubarinova-Kochina [14].

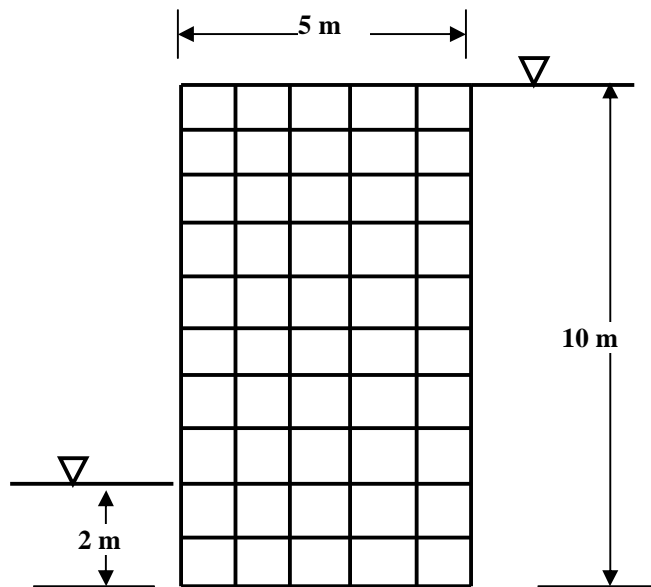


Figure (1). Mesh of the verified problem.

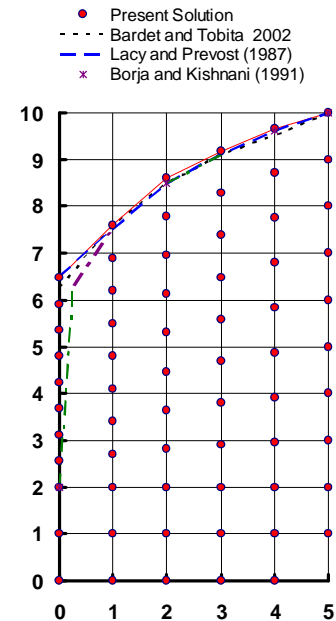


Figure (2). Comparison of free surface in a rectangular dam with the previous solutions.

4. Earth dam problem

The cases study represents free surface flow through earth dam with slopes at the two sides. The base of the dam was 400 m and the crest width was 20 m Figure (3). Upstream pressure head was 100 m and tail-water was changed with each problem. Total elements used to simulate the problem were 400 elements 40 in x direction and 10 in y direction. The shape of element was trapezoid with four nodes. The earth dam consists of shell at the upstream and downstream side and core between them. The results of the flow analysis of the dam are manifested by the permeability property. The dam is homogeneous when the permeability of whole media of dam is the same.

The two core locations studied were vertical core and core inclined towards upstream sides as will be shown in the next sections. The selection of cases was performed according to their practical advantages. For instance, the central core provides the most simple and economical choice and inclination towards upstream or downstream was to give stronger to the dam. In case of defects in the foundation along the axis of the dam, a sloping core type of dam may be indicated. The advantage of a sloping core is that it permits placement of rock fill in the large downstream portion of the dam ahead to the core, which is a major construction advantage[1].

Range of materials may be used for core while the shell has same materials for all cases. The study focused on studying the effect of core permeability. A set of permeability beginning from 0.00002 to 0.001 cm/s was selected representing different materials such as clay. Table (1) shows the values of permeability used in the case study. The ratio of core permeability to shell permeability k/k_c is also presented in Table (1). The greatest ratio is one for homogeneous dam and no ratio less than one because it is not acceptable in dam design. For the case of no drainage the core material "Compacted clay and other fine-grained" core materials permeability is less than 0.0001 cm/sec, while clean gravel give permeability greater than 0.1 cm/sec .i.e. rapid draw down. Materials with permeability between 0.0001cm/s and 0.1 cm/s are categorized as semi-pervious material such as sand and sand-gravel mixer [1].

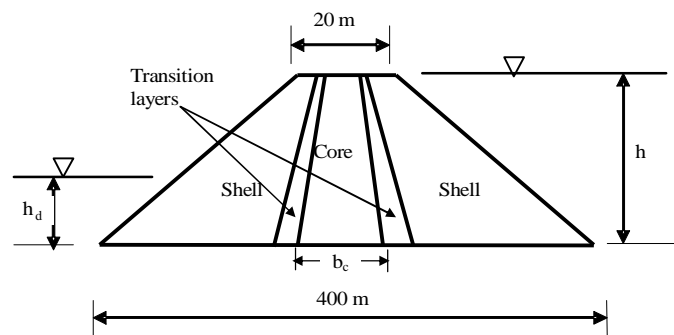


Figure (3).Schematic view of the dam problem (not to scale).

Table (1).Permeability of both shell and core that used in seepage reduction.

Item No.	K (cm/s)	k_c (cm/s)	k/k_c
1	0.001	0.00002	50.00
2	0.001	0.00003	33.33
3	0.001	0.00004	25.00
4	0.001	0.00006	16.67
5	0.001	0.0001	10.00
6	0.001	0.0002	5.00
7	0.001	0.0004	2.50
8	0.001	0.0006	1.67
9	0.001	0.0008	1.25
10	0.001	0.001	1.00

5. Vertical core

This section describes the earth dam problem and presents the discussion and analysis of the computer program results. The problem considered was a two dimensional flow through earth dam with central core. The general characteristics of the problem such as upstream head, crest width, downstream, upstream slop, and core base thickness are the same as aforementioned details. Symbols k and k_c represent permeability of shell and permeability of core respectively. h_d , and h represent downstream and upstream head respectively. Two cases of core base thickness(b_c) were used 40m and 80m and the top width of the core was 8 m. The base core thicknesses were 0.4 h and 0.8 h which are commonly used in design of dams. Figure (4) shows the schematic view of the considered problem.

In all figures presented here the quantity of seepage (q) is normalized by both upstream head (h) and shell permeability (k). Figure (5) shows the relationship between normalized quantity of seepage (q/kh) and ratio of shell permeability to core permeability (k/k_c). They were mapped for four values of ratio h_d/h (0.1, 0.2, 0.3 , 0.4). As the ratio of k/k_c increases, q/kh decreases until it reaches 0.05861 $m^3/s/m$ at $k/k_c =25$ for the case of $b_c=80m$ Table (2). After that the reduction in q/kh is 0.04620 which represents insignificant reduction. It is obviously noticed that k/k_c between 1-25 shows high effect on the q/kh for case of $b_c=80 m$ and $b_c=40m$ and insignificant effect on q/kh for $k/k_c >25$, Figure(5). It also can be noticed that ratio of h_d/h has slight effect on q/kh .

Normalized quantity of seepage (q/kh) for the case of dam with core base thickness $b_c=40$ is greater than q/kh for dam with core base thickness $b_c=80 m$ Figure (5). Certainly when the core permeability is similar to shell permeability i.e. $k_s/k_c=1$, the earth dam being homogeneous dam. As the ratio of permeability increases (k/k_c) i.e. the core permeability (k_c) decreases, the difference between the q/kh for the two cases of thickness will be appear clearly.

Figure (6) shows the relation between Q_r^n and k/k_c for four ratios of h_d/h (0.0, 0.1, 0.2, 0.3). Q_r^n is defined as the ratio of difference between q/kh for case of $b_c=40$ and that for case of $b_c=80$ to q/kh for case of $b_c=40$. It is mathematically expressed as follow:

$$Q_r^n \% = \frac{Q_{b_c=40}^n - Q_{b_c=80}^n}{Q_{b_c=40}^n} \times 100 \quad (4)$$

Q_r^n increases linearly at low ratios of k/k_c ranged between 1 and 5. The curves show low raise in Q_r^n as k/k_c increases more than 5. Maximum Q_r^n is noticed at $k/k_c=16.67$ and reduction in Q_r^n is noticed at $k/k_c >16.67$. Maximum Q_r^n was 32% for case of $h_d/h=0.0$, 30.6 for $h_d/h=$

0.1, 32.7% for $h_d/h=0.2$, and 31.4 for $h_d/h=0.3$. Average value of $(Q_r^n)_{\max}$ for the four cases of h_d/h was 31.7. Q_r^n at $k/k_c=50$ was 27% for case of $h_d/h=0$, 23.5%, for $h_d/h=0.1$, 21.6% for $h_d/h=0.2$, and 23.3% for $h_d/h=0.3$. Average value of $(Q_r^n)_{\max}$ for the four cases of h_d/h was 23.9. This means that thickness of core base has a great effect on the quantity of seepage depending essentially on the core permeability.

In addition to previous presented results, lines of seepage, free surface and surface of seepage were mapped for every case. Because of space limitation three meshes of computed free surface were presented Figure (7a,b,c). Free surface represents the upper stream line in flow domain where the pressure is atmospheric pressure. The free surface separates the saturated soil from that no flow occurs. The determination of locus of free surface is important since it can locate the surface of seepage i.e. the point of flow through downstream side. High variability in pressure is found through the core and the free surface fall down directly after the downstream face of core in the earth dam.

Referring again to Figure (7a) which shows the computed free surface for earth dam without core, one may see long surface of seepage (a). Changing the point of flow in the downstream side is required to reduce the danger of erosion which may occur in this region. Thus the benefit of using core and the importance of study such problems are clear. From computed free surface Figure (7b and c) where $b_c=40$ m it is shown that surface of seepage (a) reduces from about 139 m for homogeneous dam to about 31.5 m and reduces to about 25.6 m for $b_c=80$ m. Figure (7c) shows the computed free surface for case of earth dam with core base $b_c=40$, $Q=6.34 \times 10^{-3}$ m³/s/m and for $b_c=80$ m $Q=4.62 \times 10^{-3}$ m³/s/m.

Figure (8) shows the relation between normalized surface seepage (a/h) against (k/k_c) for case of $b_c=40$ and $b_c=80$ m. Normalized seepage surface, a/h , is equal 1.5 at $k/k_c=1$ which represents the seepage through homogeneous earth dam. This value is greater than that calculated from charts which have been presented by Gil boy [3] or from the equations that have been presented by Stello [6]. The difference may be attributed to some factors have not been included in charts or equations such as crest width, upstream head and large dam. The length of surface seepage, a/h , for the case of earth dam with core base $b_c=40$ at $k/k_c=33$ is reduced to 0.25 for $h_d/h=0.0$. It is slightly greater than that for case of $b_c=80$ m where $a/h=0.2$. It may be concluded that increasing the width of core is insignificant with respect to reducing surface of seepage. The toe drain size that can be used to drain the seepage depends on the determination of point of flow through downstream side. Using core with $b_c=40$ gives $a=31.5$ m at $k/k_c=50$ and $a=52$ m at $k/k_c=16.6$. Using core with $b_c=80$ m gives $a=25.6$ m at

$k/k_c = 50$ and $a = 41.5$ m at $k/k_c = 16.6$. The cost may determine increasing width of core or increasing the size of toe drain.

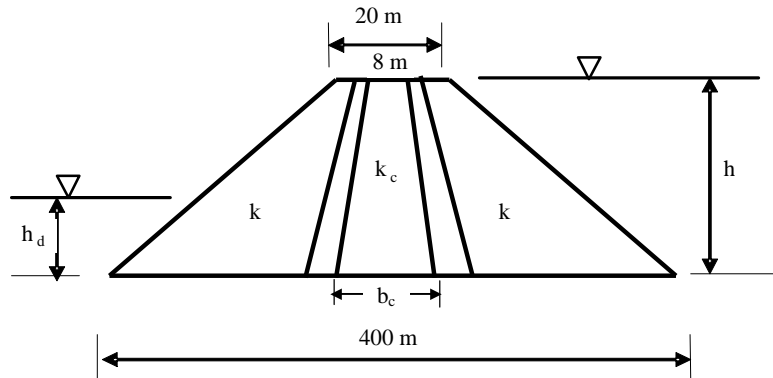


Figure (4). Schematic view of the earth dam problem.

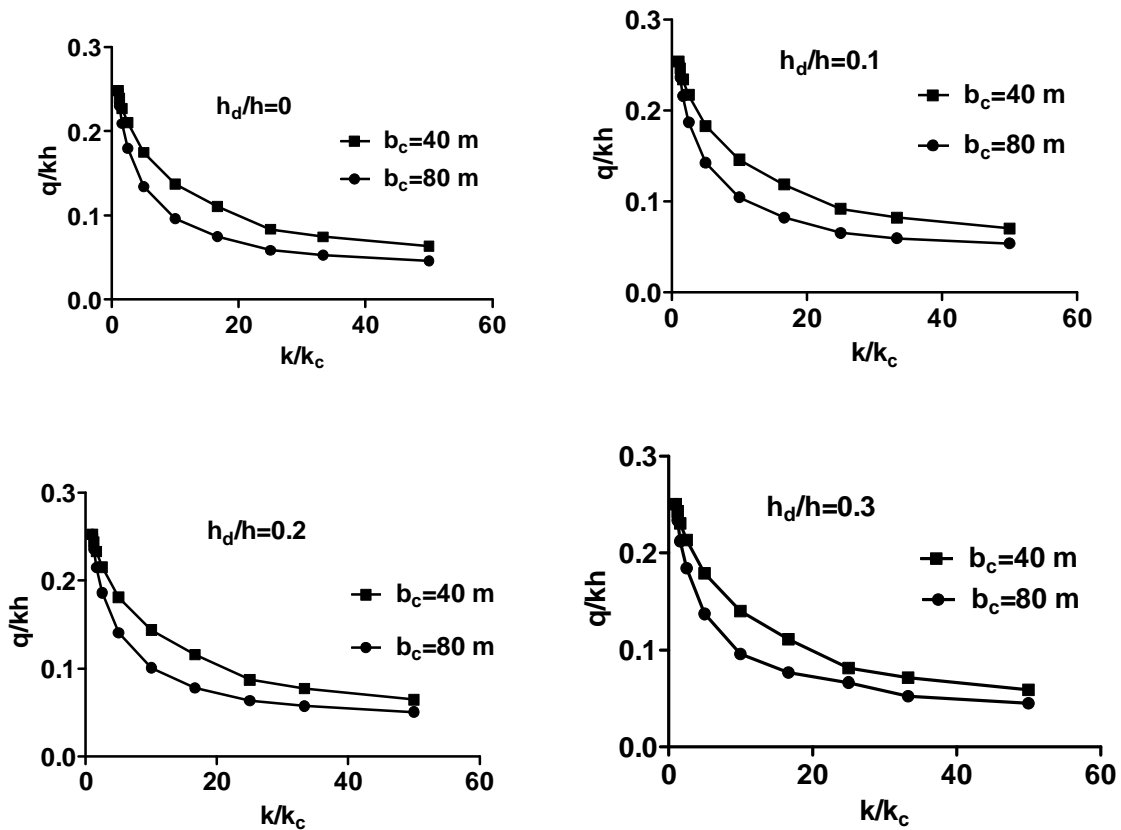


Figure (5). Normalized quantity of seepage (q/kh) versus normalized permeability (k/k_c) for ratio of downstream head to upstream head (h_d/h) 0, 0.2, 0.2, 0.3.

Table (2). q/kh for vertical core with $b_c=80$ m and $b_c=40$ m.

k/k_c	q/kh							
	$h_d/h=0.1$		$h_d/h=0.2$		$h_d/h=0.3$		$h_d/h=0.4$	
	$b_c=40$ m	$b_c=80$ m	$b_c=40$ m	$b_c=80$ m	$b_c=40$ m	$b_c=80$ m	$b_c=40$ m	$b_c=80$ m
1	0.24840	0.24840	0.25370	0.25320	0.25250	0.25220	0.25080	0.25060
16.7	0.11030	0.07493	0.11860	0.08235	0.11570	0.07785	0.11100	0.07673
25	0.08358	0.05861	0.09186	0.06574	0.08706	0.06352	0.08147	0.06654
50	0.06343	0.04620	0.07039	0.05383	0.06458	0.05064	0.05904	0.04531

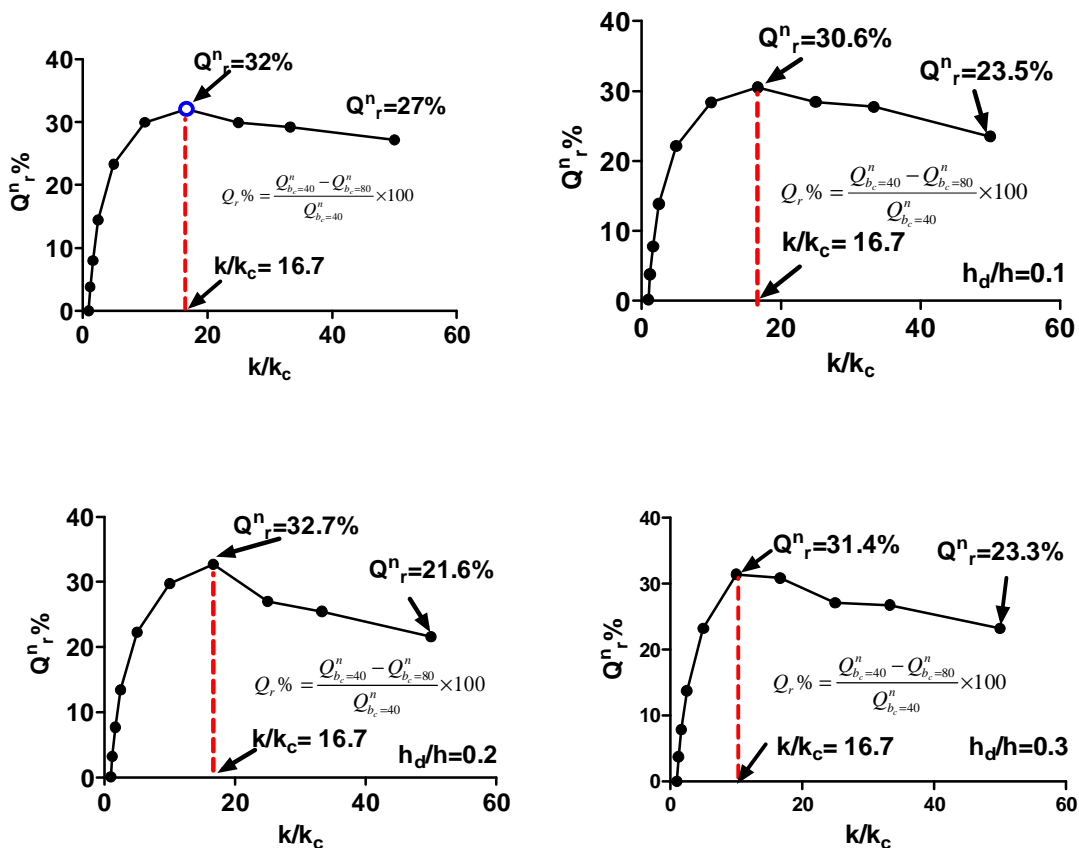


Figure (6). Relation between quantity reduction of seepage ($Q_r\%$) versus normalized permeability (k/k_c) for downstream head ratio (h_d/h) 0.1, 0.2, 0.3.

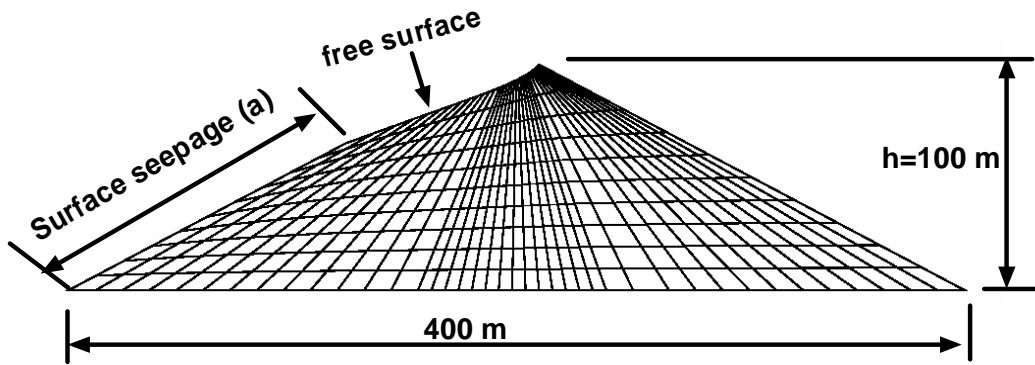


Figure (7a). Computed free surface for case of $k/k_c=1$. $Q=0.02484 \text{ m}^3/\text{s}$, $h_d/h=0$. $b_c=40$.

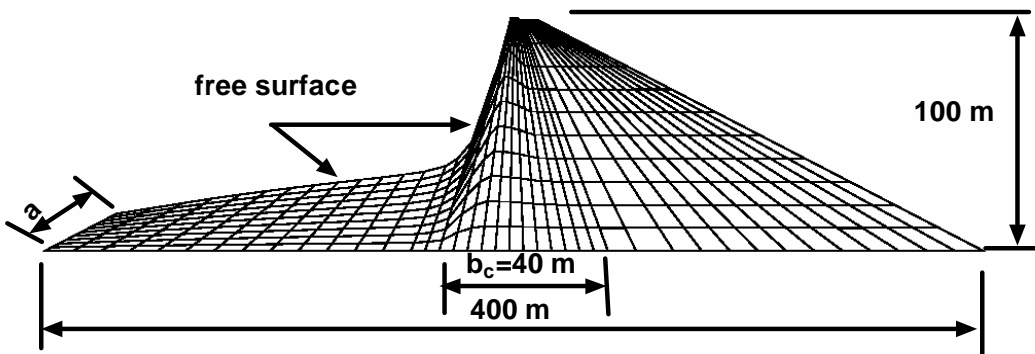


Figure (7b). Computed free surface for case of $k/k_c=50$. $Q=6.34\text{E-}03 \text{ m}^3/\text{s}$, $h_d/h=0$. $b_c=40\text{m}$.

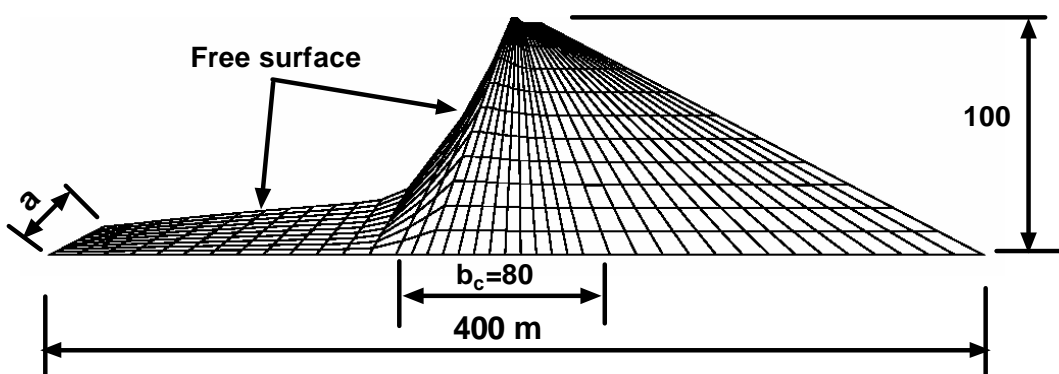


Figure (7c). Computed free surface for case of $k/k_c=50$. $Q=4.62\text{E-}03\text{m}^3/\text{s}$, $h_d/h=0$. $b_c=80$.

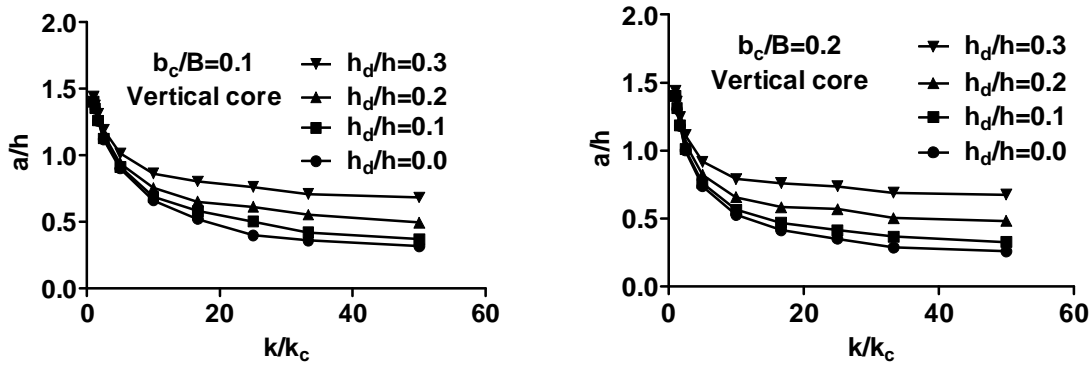


Figure (8). Normalized surface of seepage (a/h) versus normalized permeability (k/k_c) for two core base $b_c=40$ m, $b_c=80$ m for earth dam with vertical core.

6. Core inclined towards upstream side

The second application was an example of earth dam constructed with core inclined to the upstream side. Figure (9) shows a sketch demonstrating the problem of earth dam. Crest width was 20m and the base of dam was 400m. Symbol (h) refers to upstream head and symbol (h_d) refers to downstream head. The vertical core in the previous example was replaced by core inclined towards upstream side.

The relationships between normalized quantity of seepage (q/kh) and ratio of shell permeability to core permeability (k/k_c) are presented for $h_d/h=0.0, 0.1, 0.2,$ and 0.3 Figure (10). As the ratio of k/k_c increases i.e. core permeability, k_c , decreases, normalized quantity of seepage (q/kh) decreases but it fairly decreases when k/k_c being greater than 25. q/kh was 0.25 for all cases of $h_d/h=0, 0.1, 0.2$ and 0.3 at $k/k_c=1$ which represents case of homogeneous dam without any core. It was also noticed that q/kh equals 0.08 at k/k_c equal to 25. The object of using impervious core was achieved at $k/k_c=25$ and no need to use more impervious core since insignificant reduction in q/kh was obtained. The reduction in normalized quantity of seepage was 100 $(0.25-0.08)/0.25=68\%$ (Table 3). In addition to that it is obvious that h_d/h has insignificant effect on the q/kh at k/k_c greater than 25 Figure (10).

Normalized quantity of seepage (q/kh) for the case of dam with core base $b_c=40$ m is greater than (q/kh) when b_c equals 80 m Figure (10). As the ratio of permeability increases i.e. The core permeability decreases the difference between the quantities of seepage for the two cases of thickness will be clear. The relation between q/kh at $b_c=40$ and the q/kh at $b_c=80$ is mapped against the k/k_c Figure(11). Maximum difference between q/kh for case of $b_c=40$ and $b_c=80$ was at $k/k_c=25$, for cases of $h_d/h=0$ and $h_d/h=0.1$. For case of water-tail

ratio $h_d/h=0.2$ and $h_d/h=0.3$, maximum Q_r^n at $k/k_c=16.67$ was 44% and it was 41% for both cases of $h_d/h=0.2$ and 0.3 . Q_r^n is reduced to about 35% as an average at $k/k_c=50$. The thickness has an effect at $k/k_c=16.67$ more than other ratios. Core base thickness b_c has an effect on the seepage in case of core inclined towards upstream greater than that of vertical core. For instance average $(Q_r^n)_{max}=43\%$ for case of core inclined towards upstream side and average $(Q_r^n)_{max}=31\%$ for case of vertical core.

Figure 12a shows the computed free surface for dam without core. The surface seepage (a) is long and quantity of seepage is $Q=2.41 \times 10^{-2} m^3/s$. Using vertical core or inclined core reduces both the quantity of seepage and the surface seepage length Figure (12b, c). For case of $b_c=80$ m, quantity of seepage decreases more than previous case ($Q=4.54 \times 10^{-3} m^3/s$) and also show significance reduction in surface seepage Figure (12c). Figure (13) shows the effect of core on the length of surface seepage (a). It is clear that surface seepage line decreases as thickness of the core increases. Although using core materials with low permeability decrease the surface seepage line, the reduction is insignificant at $k/k_c > 33$. Using core with $b_c=40$ m give $a=40.9$ m at $k/k_c=50$ and $a=72.4$ m at $k/k_c=16.6$. Using a core with $b_c=80$ m gives $a=27.2$ m at $k/k_c=50$ m and $a=42.4$ m at $k/k_c=16.6$. The case of core base $b_c=40$ m gives greater value of (a) compared to case of the vertical core.

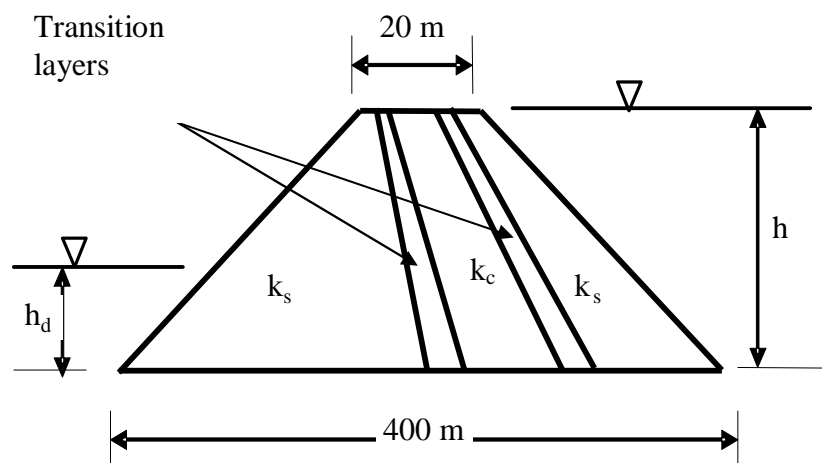


Figure (9). Sketch of dam with core inclined core to the upstream side.

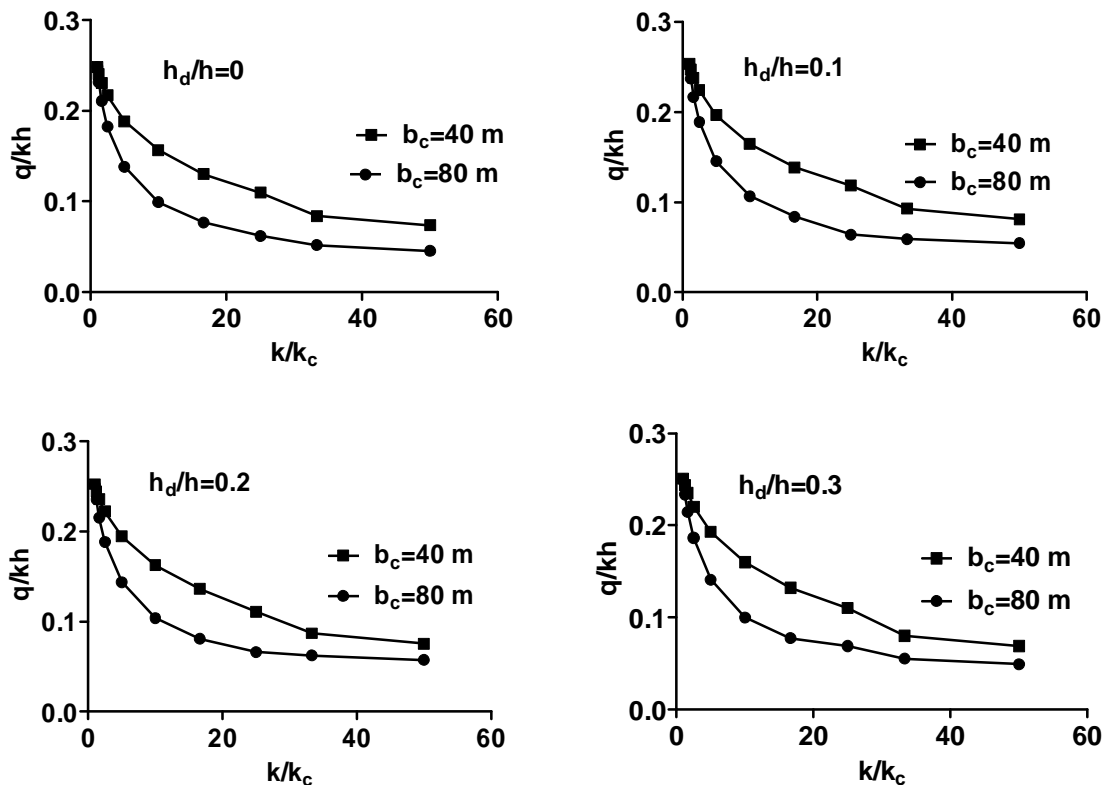


Figure (10) . Relation between normalized quantity of seepage (q/kh) versus normalized permeability (k/k_c) for downstream head (h_d/h) 0,0.2, 0.2, 0.3. (core inclined towards upstream side).

Table (3). Percent of reduction in q/kh for different k/k_c and b_c and for dam with vertical core and dam with inclined core.

k/k_c	$(Q_{k/k_c=1} - Q_{k/k_c}) / Q_{k/k_c=1}$ (Vertical Core)										
	$h_d/h=0$		0.1		0.2		0.3		Average		
	$b_c = 40$	$b_c = 80$	$b_c = 40$	$b_c = 80$	$b_c = 40$	$b_c = 80$	$b_c = 40$	$b_c = 80$	$b_c = 40$	$b_c = 80$	
16	69.83	55.60	67.48	53.25	69.13	54.18	69.38	55.74	68.95	54.69	
25	74.46	76.41	72.25	74.04	74.42	74.81	76.46	73.44	74.39	74.67	
50	81.40	74.46	78.74	72.25	79.91	74.42	81.91	76.46	80.49	74.40	
k/k_c	$(Q_{k/k_c=1} - Q_{k/k_c}) / Q_{k/k_c=1}$ (Inclined Core)										
	16	69.09	47.53	66.68	45.31	67.88	46.041	68.92	47.27	68.14	46.54
	25	55.89	75.11	53.31	74.56	56.05	73.65	56.04	72.41	55.32	73.93
	50	81.74	70.39	78.39	67.94	77.09	70.08	80.30	72.44	79.38	70.21

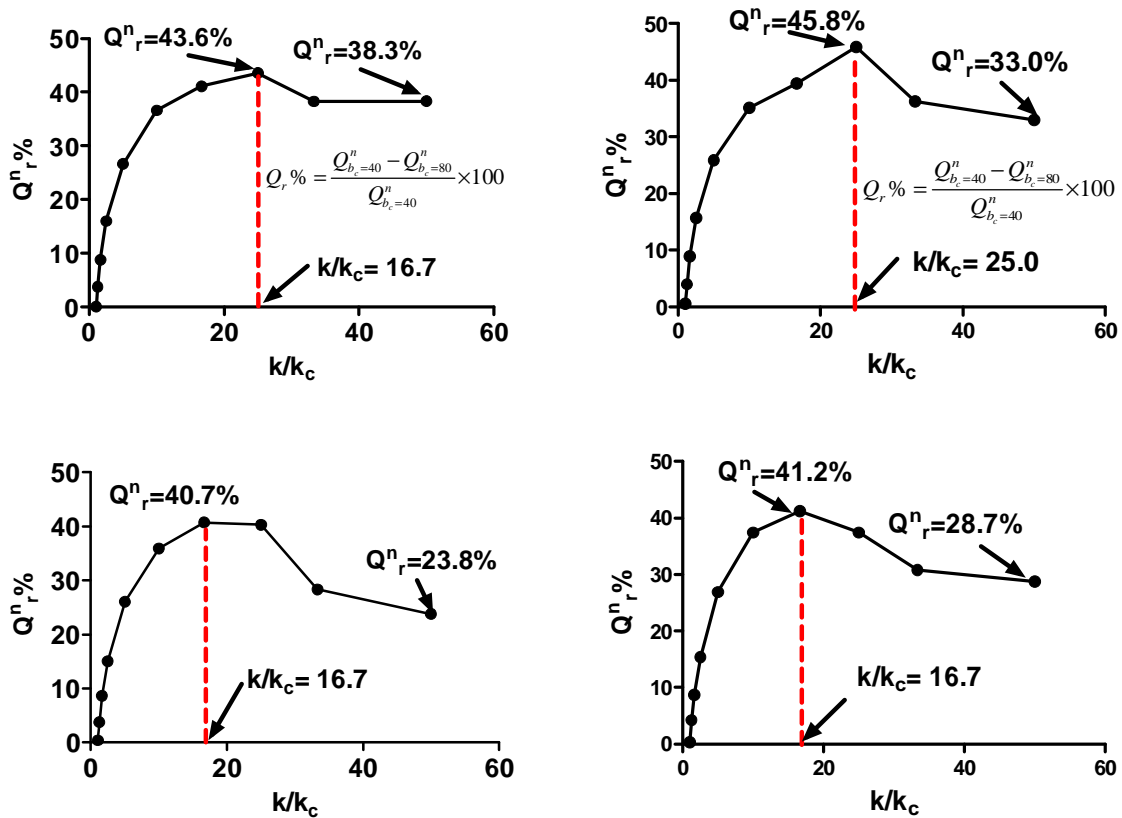


Figure (11). Relation between quantity reduction of seepage ($Q_r\%$) versus normalized permeability (k/k_c) for downstream head ratio (h_d/h) 0,0.2, 0.2, 0.3. (core towards upstream side).

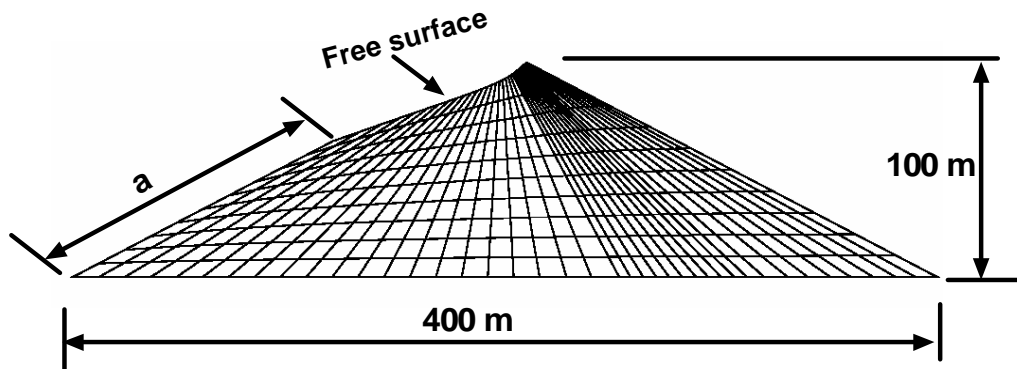


Figure (12a). Computed free surface for case of $k/k_c=1.25$. $Q=2.41E-02m^3/s$, $h_d/h=0$. $b_c=40m$.

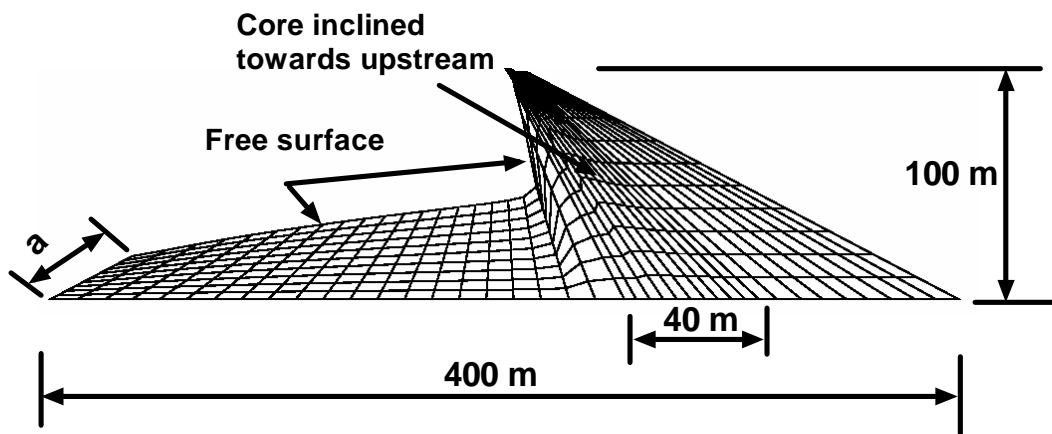


Figure (12b). Computed free surface for case of $k/k_c=50$. $Q=1.91E-04 \text{ m}^3/\text{s}$, $h_d/h=0$. $b_c=40\text{m}$.

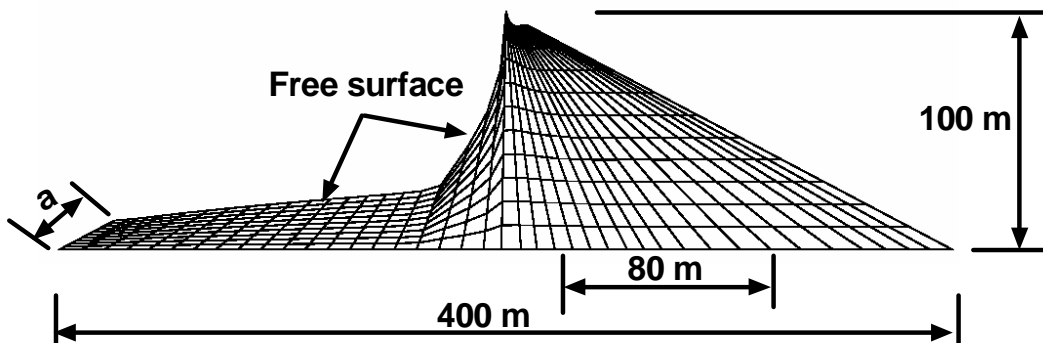


Figure (12c). Computed free surface for case of $k/k_c=50$. $Q=4.54E-03\text{m}^3/\text{s}$, $h_d/h=0$. $b_c=80$.

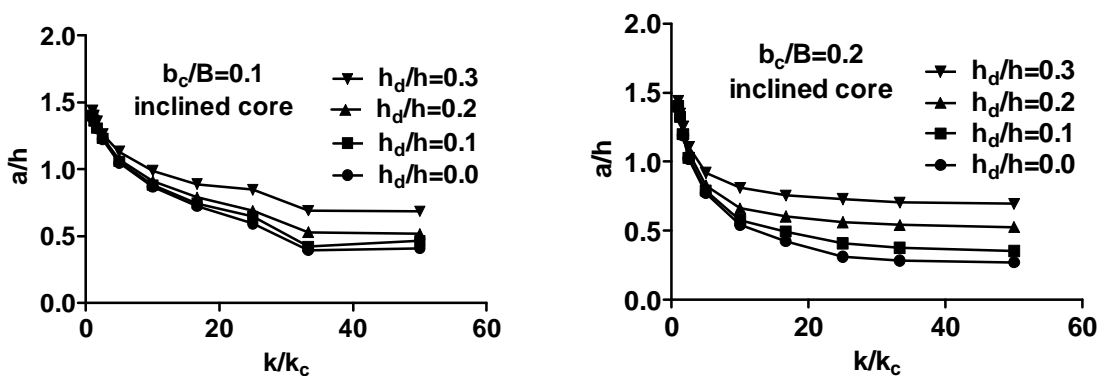


Figure (13). Normalized surface of seepage (a/h) versus normalized permeability (k/k_c) for two core base $b_c=40 \text{ m}$, $b_c=80 \text{ m}$, for earth dam with core inclined towards upstream side.

7. Conclusions

Finite element method was used to solve the Laplace equation and to locate the free surface. Verifying the results of the computer program of flow in two dimensional (p 7.3) [10] with available methods gives a good agreement. Two cases of core were studied: a vertical core and a core inclined towards the upstream side. The study demonstrated that the quantity of seepage reduces as core with low permeability is used. It was found that using vertical core with core base thickness, $b_c=40\text{m}$, and with permeability 6% of shell permeability decreases the normalized quantity of seepage (q/kh) to 0.45 that obtained from flow through homogeneous dam and using vertical core with permeability 2% i.e. $k/k_c=50$ of shell permeability decreases the normalized quantity of seepage (q/kh) to 0.25 that obtained for homogeneous dam.

The core base thickness also affects the quantity of seepage. It was found that doubling the base width decreases the quantity of seepage to about 68-76 %. The location of the cores lightly influences the quantity of seepage. It was found that using core inclined towards upstream increases the quantity of seepage 13%.

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9.Nomenclature

List of symbols and variables

Symbol or variable	definition
A	Surface seepage
b_c	Core width
k	Permeability
kc	Permeability of core
h	Upstream head
Q	Quantity of seepage
hd	Downstream head
Q^n	q/kh
$Q_{bc=40}^n$	q/kh for case of core width = 40 m
$Q_{bc=80}^n$	q/kh for case of core width = 80 m
$Q_r^{n\%}$	$Q_r^n \% = \frac{Q_{bc=40}^n - Q_{bc=80}^n}{Q_{bc=40}^n} \times 100$