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Reduction of Peak to Average Power Ratio in OFDM System Using Discrete Cosine Transform Technique

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Abstract

Orthogonal frequency division multiplexing (OFDM) is a form of multi-carrier modulation technique with high spectral efficiency, robustness to channel fading and immunity to impulse interference. Recently, it is being used for both wireless and wired high data rate communications. Despite of its many advantages, OFDM has a main drawback, namely high Peak to Average Power Ratio (PAPR).High PAPR causes saturation in power amplifiers, and leads to inter-modulation products among the sub carriers and disturbing out of band energy. Therefore, it is desirable to reduce the PAPR. In this paper we used Inverse Discrete Cosine Transform (IDCT) technique in the transmitter side to reduce PAPR problem. The use of this technique in the output of Inverse Fast Fourier Transform (IFFT) stage gave good results toward reduction PAPR. The receiver side includes Discrete Cosine Transform to achieve correct detection. Computer simulation tests have been applied on IEEE 802.11a standard as OFDM practical system with the proposed method .The results exhibit the ability of such techniques to reduce the PAPR with no major effect on the system performance as compared with the conventional OFDM technique.

Keywords: OFDM, Peak to Average Power Ratio (PAPR) ,IEEE 802.11a, Rayleigh Fading channel, Discrete Cosine Transform, Additive White Gaussian Noise (AWGN).

المـستخلـص

يُعد الإرسال المتعدد بتقسيم التردد المتعامد OFDM تقنية عالية الكفاءة باستغلال الطيف الترددي لهيئة التضمين متعدد الحامل و مقاوم إلى خفوت القناة والحصانة من التداخل. حديثا, استخدم نظام OFDM للإرسال اللاسلكي وللاتصالات السلكية الرقمية ذات معدل الإرسال العالي. على الرغم من المحاسن الكثيرة للنظام, OFDM يمتلك عيب رئيسي هو ارتفاع نسبة أعلى إلى معدل القدرة PAPR. ارتفاعPAPR يجعل المكبرات تعمل بمنطقة التشبع , مما يؤدي إلى التداخل بين النواقل الفرعية في النطاق الترددي و الإرسال الغير المرغوب به خارج النطاق المحدد. لذلك أصبح من

المرغوب به تقليل ارتفاع نسبة أعلى إلى معدل القدرة PAPR. في هذا البحث استخدمنا تقنية معكوس تحويلة الجيب تمام المجزئة IDCT في جهة الإرسال لتقليل مشكلة PAPR في نظام OFDM. ان استخدام هذه التقنية من بعد إخراج معكوس تحويلة فورير السريعة أعطت نتائج جيدة باتجاه تقليل PAPR. جهة الاستقبال تضمنت استخدام تحويلة الجيب تمام المجزئة DCT للكشف الصحيح للبيانات. تمت محاكاة النظام المقترح ضمن نظام IEEE 802.11a كنظام معتمد عمليا باستخدام الحاسوب . النتائج أظهرت قدرة النظام المقترح لتقليل ارتفاع PAPR بدون حدوث أي تأثير رئيسي على أداء النظام المقترح بالمقارنة مع نظام OFDM التقليدي.

1. Introduction

Orthogonal Frequency Division Multiplexing (OFDM) is a multicarrier transmission technique which is widely adopted in different communication applications. OFDM prevents Inter Symbol Interference (ISI) by inserting a guard interval and mitigates the frequency selectivity of a multi-path channel by using a simple equalizer. This simplifies the design of the receiver and leads to inexpensive hardware implementations. OFDM has been employed in diverse wired and wireless applications. For instance, in digital audio and video broadcasting[1], digital subscriber lines using discrete multi-tone [2], the wireless LAN systems such as, IEEE802.11, HIPERLAN and MMAC[3], wireless broadband service[4] and also is a strong candidate for next generation cellular systems[5].OFDM systems have the inherent problem of high Peak to Average Power Ratio(PAPR). Peak power level of an OFDM signal is much higher than the average power due to addition of large number of subcarriers in phase. A large PAPR brings disadvantages like an increased complexity of the Analog – to – Digital (A/D) and Digital – to – Analog (D/A) converters and a reduced efficiency of the RF power amplifier. There have been two sorts of approaches to deal with PAPR of OFDM, one includes amplitude clipping [6], clipping and filtering [7], coding [8], active constellation extension (ACE) [9]; and the other one, which can be regarded as multiple signal representation technique, contains partial transmit sequence (PTS) [10], selected mapping (SLM) [11], erasure pattern selection (EPS) [12] and interleaving [13]. The latter type is also called probabilistic method, and attracts most of the attention. The character of this kind of methods is not to eliminate PAPR completely, but to reduce the probability of its occurrence.

In this paper, we proposed a technique to reduce PAPR in OFDM system using Inverse Discrete Cosine Transform IDCT scheme. The kernel concept of the proposed scheme is based on the ability of IDCT transform to rearrange the energy of signal in closed levels in time domain ,and the signal completely de-correlated in transform domain [14]. At the same time, the proposed scheme greatly reduce the Peak to Average Power Ratio and the system as character of low complexity hardware and without transmitted side information.

2. Fundamentals of OFDM system

An OFDM symbol consists of N subcarriers by the frequency spacing of Δf. Thus, the total bandwidth B will be divided into N equally spaced subcarriers and all the subcarriers are orthogonal to each other within a time interval of length $T=1/\Delta f$ .Each subcarrier can be modulated independently with the complex modulation symbol $X_{m,n}$, where m is a time index and n is a subcarrier index. Then within the time interval T the following signal of the m-th OFDM block period can be described by equation [15]:

$$x_m(t) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} X_{m,n} g_n(t - mT) \tag{1}$$

Where, $g_n(t)$ is:

$$g_n(t) = \begin{cases} exp(j2\pi n\Delta f), & 0 \le t \le T \\ 0 & \text{elsewhere} \end{cases}$$

The total continuous time signal x(t) consisting of all the OFDM blocks is given by:

$$x(t) = \frac{1}{\sqrt{N}} \sum_{m=0}^{\infty} \sum_{n=0}^{N-1} X_{m,n} g_n(t - mT) \tag{2}$$

Consider a single OFDM symbol, since m=0 and $X_{m,n}$ can be replaced by X_n .Then, the OFDM signal can be described as follows:

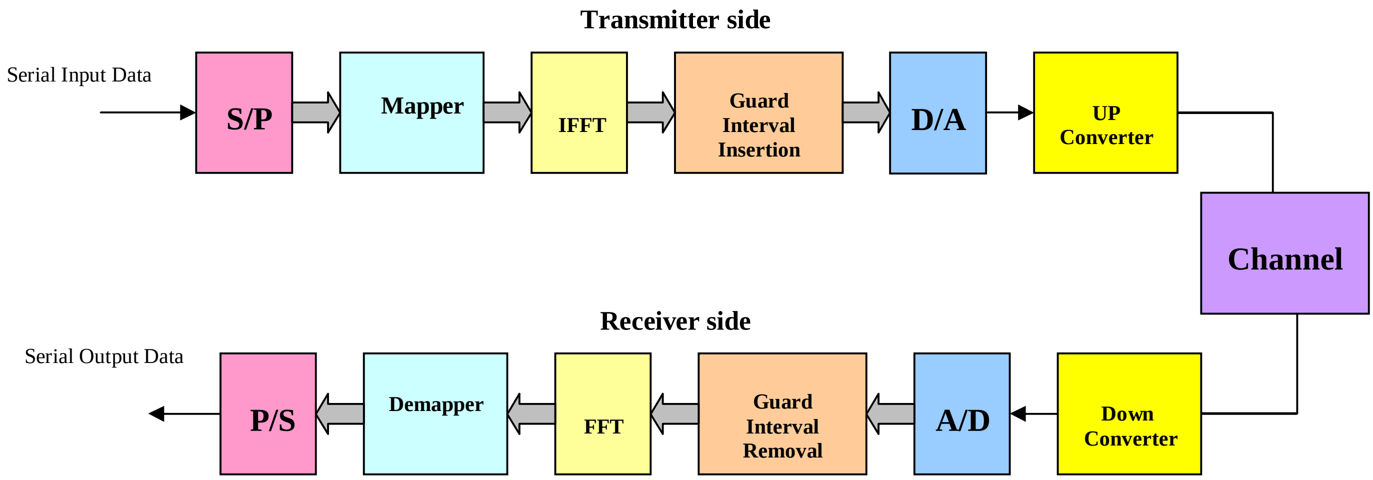
$$x(t) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} X_n e^{j2pn\Delta ft} \tag{3}$$

If the bandwidth of the OFDM signal is $B=N\times\Delta f$ and the signal $x(t)$ is sampled by the sampling time of $\Delta t=1/B=1/(N\Delta f)$, then the OFDM signal in discrete time form can be written as:

$$x_k = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} X_n e^{j2pkn/N} \qquad k= 0, 1, 2,\ldots\ldots\ldots, N-1 \qquad (4)$$

where, n denotes the index in frequency domain and X_n is the complex symbol in frequency domain. Furthermore, equation (4) can be expressed using the IFFT [16].

Figure (1). shows a typical system block diagram of an Basic OFDM system. The serial input data stream is converted to N parallel subchannels and mapping with a selected modulation scheme, resulting in N subchannels containing information in complex number form.



Figure(1). Block diagram of the basic OFDM system.

These complex values are then sent to the N channel IFFT. The parallel signals are converted back to a serial sequence by using a P/S device. A guard interval is inserted to reduce the effect of ISI caused by multipath propagation. Finally, the signal is converted to analogue signal and converted back up to a form suitable for transmission. At the receiver, a reverse procedure is used to demodulate the OFDM signal.

3. PAPR in OFDM system

Consider an OFDM with N sub carriers. Each OFDM block(OFDM symol), x(t), $0 \le t \le T$, consists of N complex base band data $x_0, x_1, x_2, \ldots\ldots\ldots, x_{N-1}$ carried on the N subcarriers respectively for a symbol period of T. According to eq.(4), the peak instantaneous power is:

$$\text{Pmax} = \max_{t\in[0,T)} |x(t)|^2 \tag{5}$$

An OFDM symbol sequence can be represented by x(t), x(t+T), ……., x(t+mT), ……
The average power of OFDM symbol sequence as following:

$$\text{Pav}(x_0, x_1, x_2, \ldots\ldots\ldots, x_{n-1}) = \tfrac{1}{N}\sum_{k=0}^{N-1} E[|x_k|^2] \tag{6}$$

Where $E[|x_k|^2]$ is the expected value of $|x_k|^2$. The PAPR of the OFDM symbol x(t) is:

$$\text{PAPR} = \frac{P_{\max}}{P_{av}(x_0, x_1, \ldots\ldots, x_{N-1})} = \frac{\max\limits_{t\in[0.T)} |x(t)|^2}{\frac{1}{N}\sum\limits_{k=0}^{N-1} E[|x_k|^2]} \tag{7}$$

If the power of input signal is standard; i. e. the $E[|x_k|^2] = 1$;then :

$$\max_{t\in[0,T)} |x(t)|^2 = \max_{t\in[0,T)} \left| \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X_k e^{\frac{j2pnk}{N}} \right|^2 \le N \tag{8}$$

As a result, the PAPR value is not larger than the number N of sub carriers, e.g. the peak power value of OFDM signals is N times larger than its average power. So, the maximum of PAPR equals to N. With the increase in the number N of sub channels, the maximum of PAPR increases linearly. This makes high demands on the linear range of the front-end amplifier in sending side.

Although the probability of largest PAPR is low, in order to transfer these high PAPR of OFDM signal with non-distortion, all the linearity of the HPA in sending side, the front-end amplifier and A/D converter should meet the high requirement. But these equipments meeting the high requirement are expensive. Therefore, it is necessary and important to reduce PAPR in OFDM system.

4. Distribution of the PAPR in OFDM system

For an OFDM signal with N subcarriers, the PAPR can be defined as [17].

$$PAPR = \frac{\max|x(t)|^2}{E|x(t)|^2} \tag{9}$$

In particular, a base band OFDM signal with N subchannels has maximum PAPR equal[18] :

$$\text{PAPRmax}=10\log_{10}(N) \tag{10}$$

For M-PSK modulation, there are only M^2 sequences having maximum PAPR equal to 10log(N) as described in [18]. This means, the number of sequences that gives very high PAPR is not very high (for BPSK M=2 then only 4 sequence from 2^N have maximum PAPR equal 10log(N)). If the number of subchannels increases, the ratio of the sequence (**R**) that gives so much PAPR and all distinct sequences decreases rapidly. The overall number of distinct sequences for the N subcarriers OFDM system with M-PSK is M^N. Thus the ratio can be obtained by equation (11) as:

$$R = \frac{M^2}{M^N} = M^{2-N} \tag{11}$$

From the central limit theorem, it follows that for large values of N (N>64), the real and imaginary values of x(t) become Gaussian distributed. Therefore the amplitude of the OFDM signal has Rayleigh distribution, with a cumulative distribution given by $F(z)= 1- e^{-z}$. The probability that the PAPR is below a threshold level can be written as:

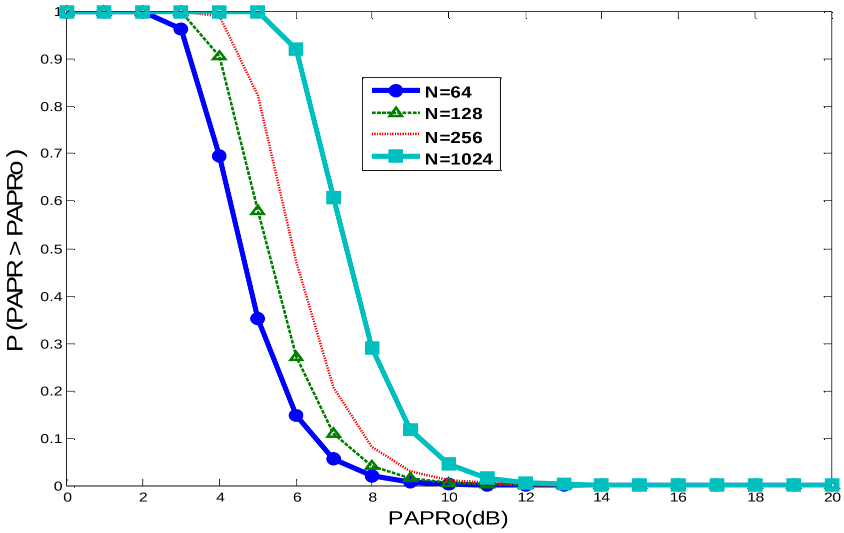
$$P(PAPR \le z) = (1 - e^{-z})^N \tag{12}$$

The **C**omplementary **C**umulative **D**istribution **F**unction (**CCDF**) of PAPR of an OFDM is usually used, and can be expressed as:

$$P\,(PAPR \ge PAPR_o) = 1 - (1 - e^{-PAPR_o})^N \tag{13}$$

Where $PAPR_o$ is the threshold power (dB).

This theoretical derivation is plotted against simulated values in Fig.(2) for different values of N.



Figure(2). CCDF for different values of subcarriers .

5. IDCT – OFDM proposed technique

Discrete Cosine Transform (**DCT)** used in most digital signal processing such as digital image processing. The most common DCT definition of a 1-D sequence of length N is:-

$$X_{DCT}(u) = a(u)\sum_{i=0}^{N-1} x(i)\cos(\frac{p(2i+1)u}{2N}) \qquad (14)$$

for u=0,1,2,…..,N-1.

Similarly, the inverse transformation IDCT is defined as:

$$x(i) = a(u)\sum_{u=0}^{N-1} X_{DCT}(u)\cos(\frac{p(2i+1)u}{2N}) \qquad (15)$$

For i=0, 1, 2,….., N-1.In both equations (14) & (15).

$$a(u)=\begin{cases}\sqrt{\dfrac{1}{N}} & \text{for } u=0\\ \sqrt{\dfrac{2}{N}} & \text{for } u\neq 0\end{cases}$$

In this paper ,Inverse Discrete Cosine Transform (IDCT) is cascaded after IFFT to insure the required redistribution power of subcarriers. The orthogonality of subcarriers in OFDM signal maintained because of the orthogonality and decorrelated property of IDCT [19]. The original signal can be resumed if DCT is inserted before FFT. DCT-OFDM system block is shown in Figure(3).

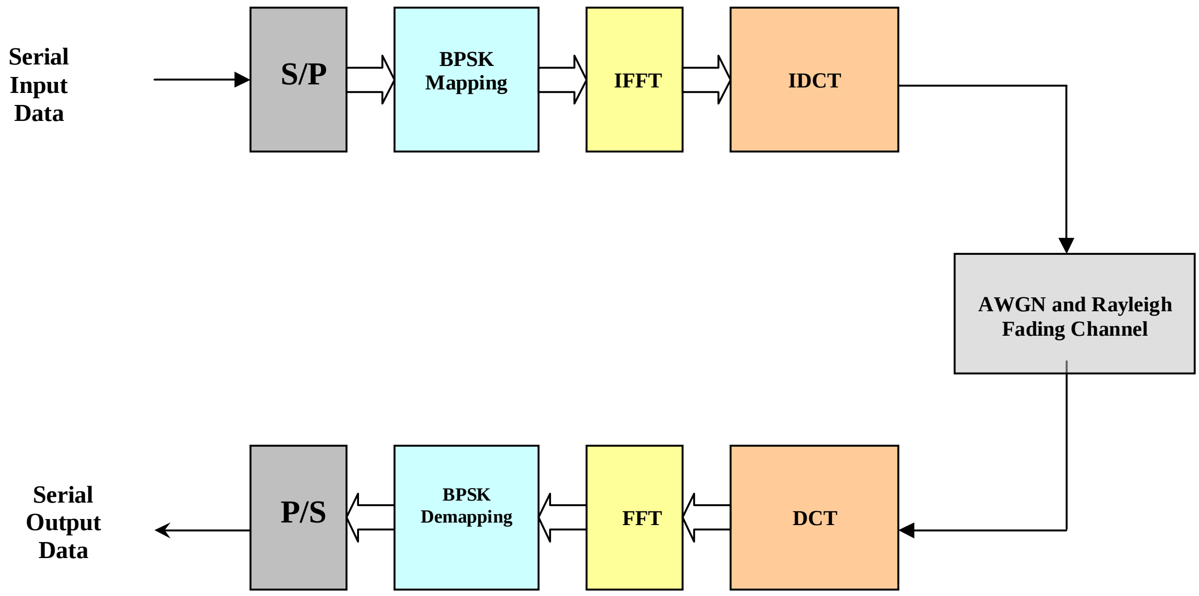
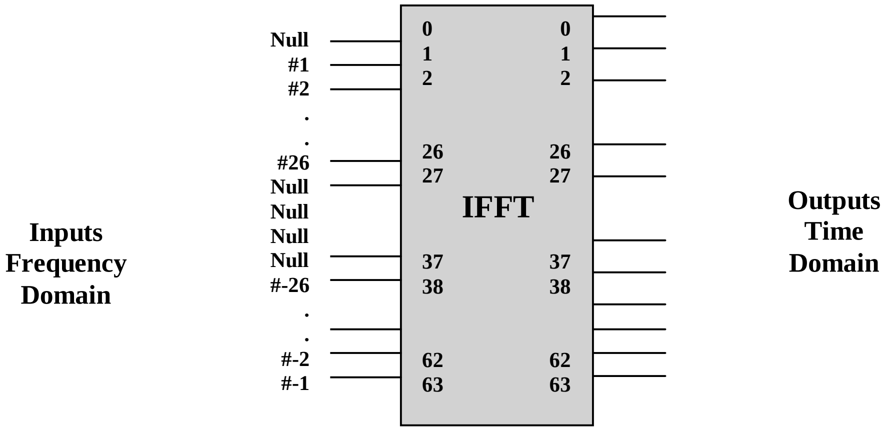


Figure (3). Block diagram of proposed scheme.

6. OFDM simulation model

The model of DCT-OFDM transmitter and receiver was made in MATLAB **7.6 (R2008a)** octave codes. For all simulated tests, the **IEEE 802.11a** as standard wireless LAN OFDM system is used with 640 K symbols.

IEEE 802.11a used as Wireless LAN communication system. The IEEE 802.11a standard specifies an OFDM physical layer (PHY) that splits an information signal across 52 separate subcarriers to provide transmission of data at a different rate. Four of the subcarriers are pilot that the system uses as a reference to disregard frequency or phase shifts of the signal during transmission. A pseudo binary sequence is sent through the pilot subchannels to prevent the generation of spectral lines. The remaining 48 subcarriers provide separate pathways for sending the information in a parallel fashion. The resulting subcarrier frequency spacing is 0.3125 MHz (for a 20 MHz) with 64 possible subcarrier frequency. Binary Phase Shift Keying used as a modulation technique. A 64-point IFFT is used, the coefficients 1 to 26 are mapped to the same numbered IFFT inputs, while the coefficients –26 to –1 are copied into IFFT inputs 38 to 63. The rest of the inputs, 27 to 37 and the 0 (dc) input, are set to zero. This mapping is illustrated in Figure (4).

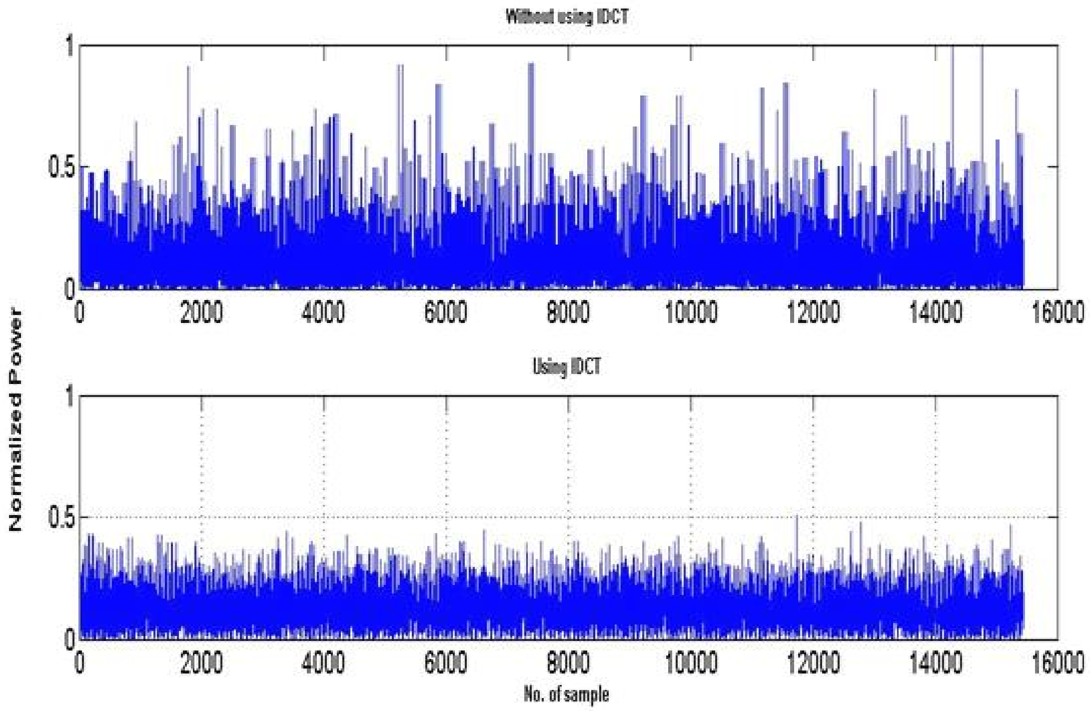


Figure(4).Inputs & outputs of IFFT in 802.11a system.

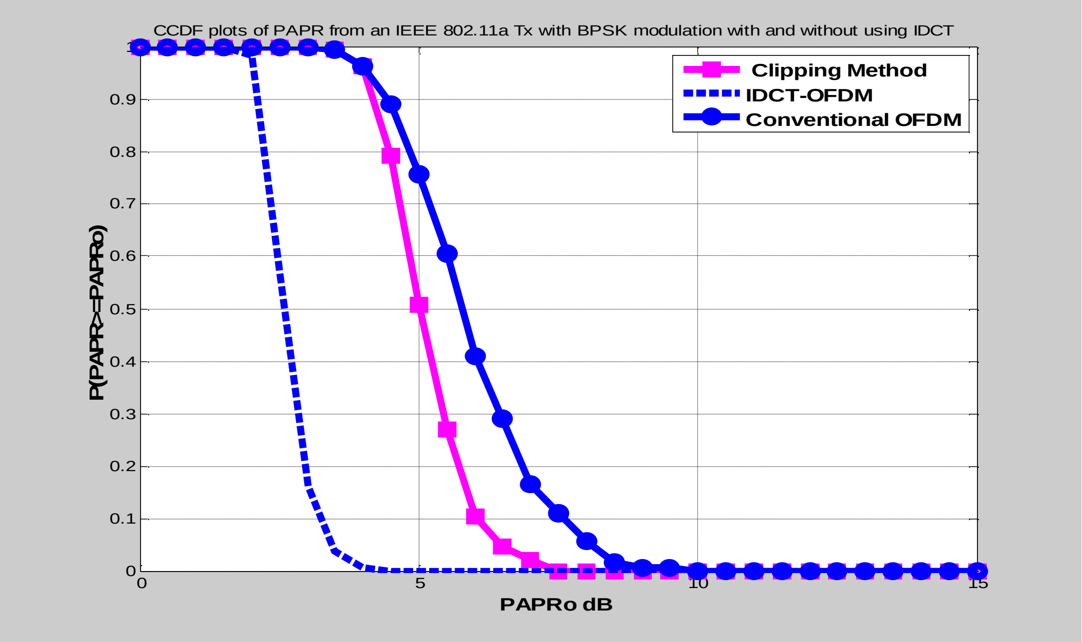
7. Results and discussion

Figure(5) shows the normalized output power of the proposed OFDM system with and without IDCT technique. From this figure, it is clear that IDCT reduces maximum power. This result because of IDCT rearranges power distribution of subcarriers.

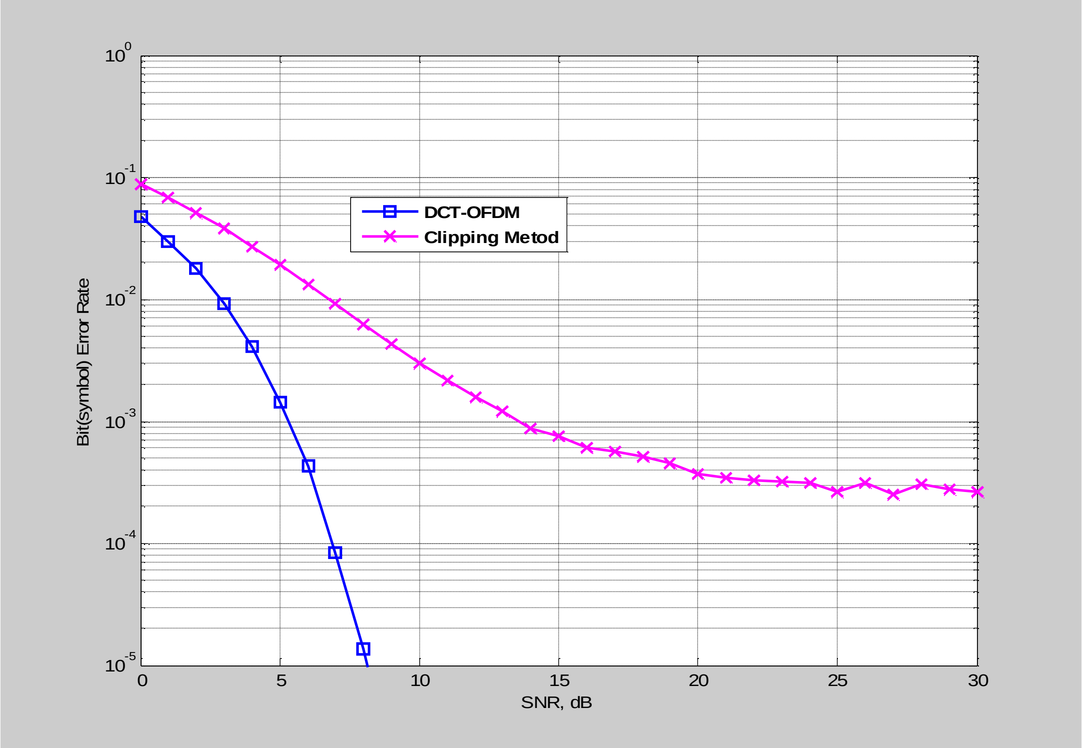
There are many methods to reduce the PAPR problem of OFDM system .Clipping method consider the easy method for reduction PAPR. We will compare PAPR performance of conventional OFDM , Clipping Method and IDCT-OFDM. The best way to compare these techniques is Complementary Cumulative Distribution Function CCDF as shown Figure(6). IDCT-OFDM exhibited more reduction in PAPR compare with Clipping method. The simulation parameters of Clipping method chosen to ensure that it gave similar BER performance of IDCT-OFDM in receiver side. From Figure(6),the PAPR of IDCT-OFDM systems can be reduced about 6dB compared with convention OFDM and about 3dB less than clipping methods.



Figure(5).Normalize power for OFDM with and without IDCT technique.



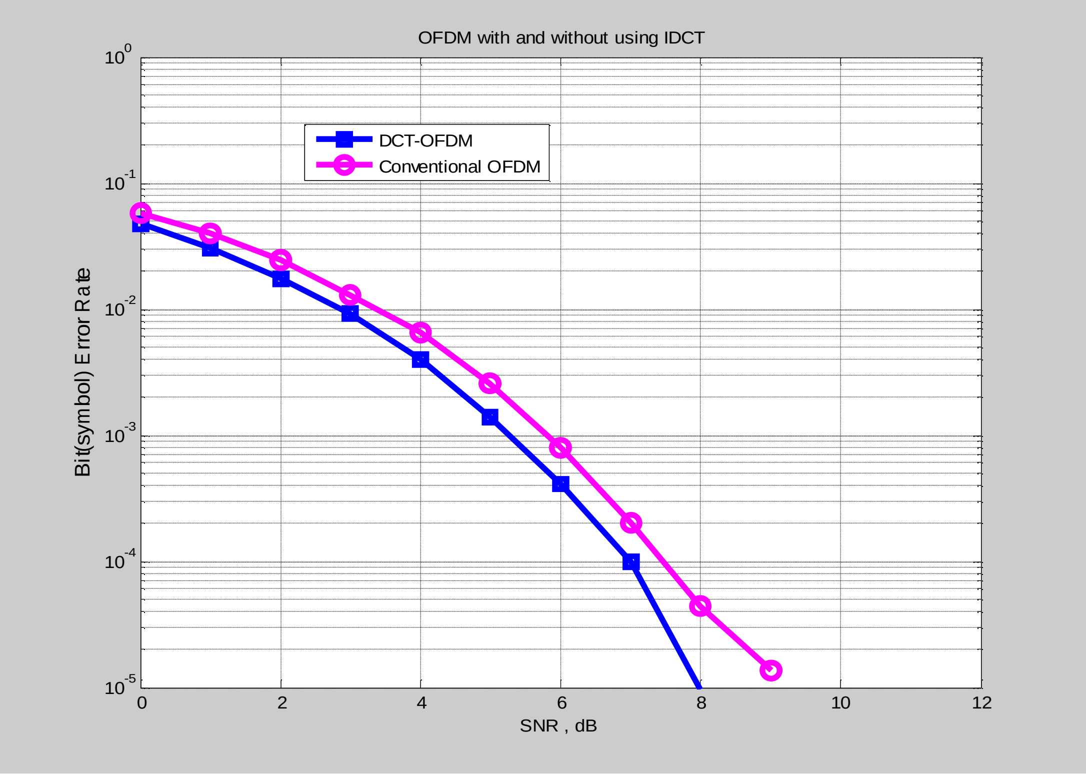
Figure(6).CCDF Comparison between clipping and IDCT.



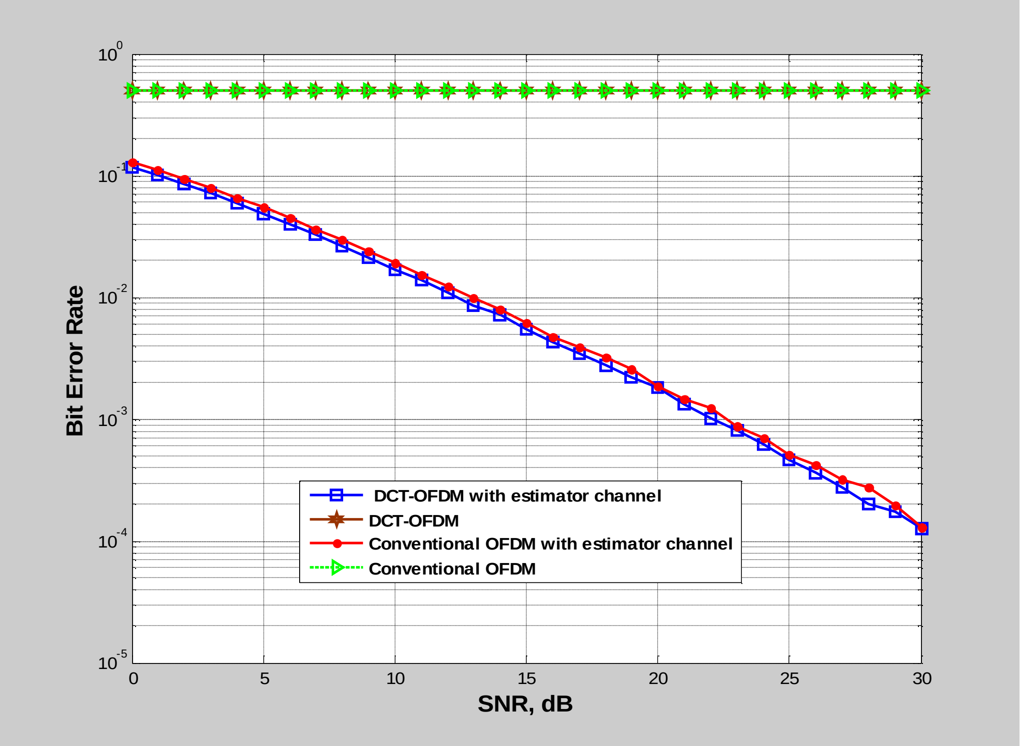
Figure(7).BER performance for IDCT and clipping method.

Figure (7) shows the BER performance for IDCT and Clipping method when the CCDF of clipping method is similar to CCDF of IDCT-OFDM method. The similarity can be achieved when the power threshold is change for suitable value where $PAPR_o$ don’t exceed 4dB. The BER in Figure(7) was under AWGN channel environments. The limitation of Clipping method because of Clipping causes distorting the OFDM signal. IDCT technique exhibited good performance toward reduction PAPR in transmitter side and decreasing Bit Error Rate (BER) in receiver side about 1dB less than conventional OFDM as shown Figure (8).This results because of de-noising property of IDCT[20].

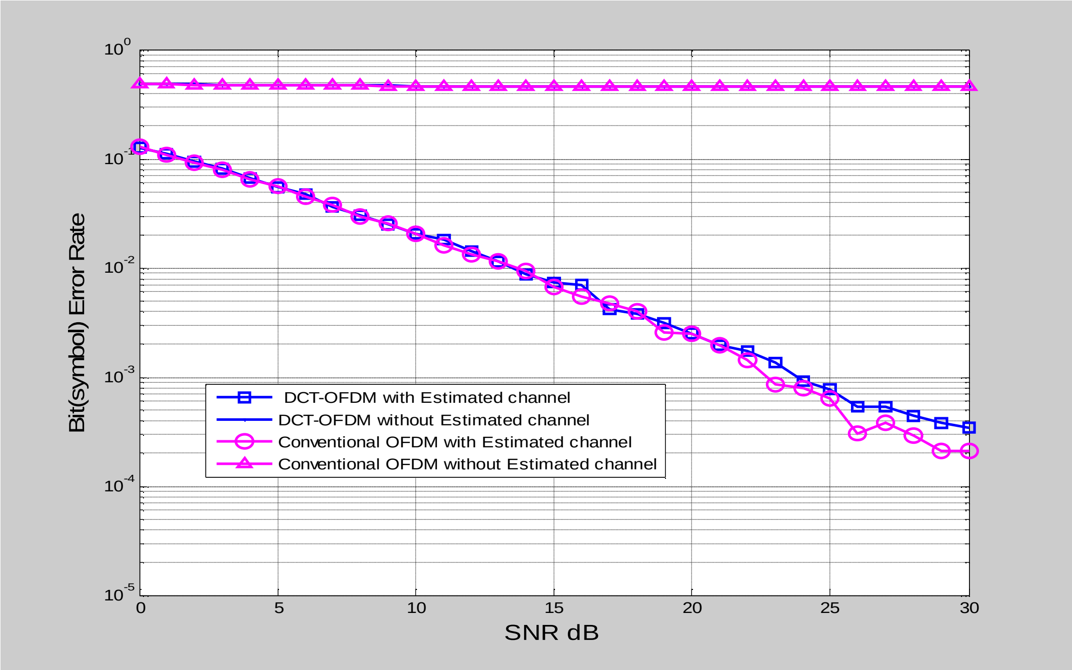
Figure(9) and Figure(10) show the BER performance of OFDM system with and without using IDCT under static flat fading and selective (2-Taps) noisy channel .It is important to see that, without channel estimation, data cannot be recovered. IDCT method has no effect on BER performance compared with conventional OFDM because of the output coefficients of IDCT orthogonal and the properties of IDCT it seemed similar to properties of FFT in ordinary OFDM.



Figure(8).BER performance of OFDM with and without IDCT in AWGN channel.



Figure(9). Performance of OFDM with and without using IDCT under Rayleigh flat fading(single-tap) noisy channel.



Figure(10). Performance of OFDM with and without using IDCT under Rayleigh selective fading(2-Taps) noisy channel.

8. Conclusions

Inverse Discrete Cosine Transform IDCT cascaded after IFFT exhibited good results toward reducing PAPR with acceptable performance as compared with clipping method and conventional OFDM. These results because energy de-compaction property for IDCT in Tx side and denoising property for DCT in Rx side. The BER performance of DCT-OFDM likes the performance conventional OFDM under static flat and selective (2-Taps) fading noisy (AWGN) channel. DCT-OFDM technique is not needed to transmit side information and is simple hardware implementation (DCT blocks available as IC chip sets). Fore these reasons DCT-OFDM makes it more applicable.

9. References

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