High Performance PCC-DMT System
Based on Hybrid Multi-Wavelet Functions

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Abstract

In this paper a scheme is proposed to improve the Bit Error Rate (BER) performance for Discrete Multi-Tone (DMT) system in wireless channel. The conventional DMT system is based on Fast Fourier Transform (FFT). The proposed system is based on Hybrid Multi-Wavelet Transform (HMWT). The HMWT is mixed between FFT and Multi-Wavelet Transform (MWT). The Inter-Carrier Interference (ICI) in DMT system due to Doppler spread is reduced by using Polynomial Cancellation Coding (PCC). Simulation results are done in three different types of channels: Additive White Gaussian Noise (AWGN) channel, flat fading channel and selective fading channel. The results show that the PCC-DMT system based on HMWT achieves better performance than the PCC-DMT system based on FFT and the PCC-DMT system based on MWT in all channels types.

Keywords: DMT, FFT, Multi-Wavelet Transform (MWT), Hybrid Multi-Wavelet Transform (HMWT), Polynomial Cancellation Coding (PCC).

منظومة PCC-DMT المتعددة عالية الأداء المستندة على تهيج تحويل الموجة المتعدد المستخلص

في هذا البحث تم اقتراح نظام لتحسين معدل الخطأ لمنظمة متعددة النغمات المنفصلة (DMT) اللاسلكية. أن نظام DMT الموجود يعتمد على تحويل فوريير السريع (FFT). أما النظام المقترح يعتمد على تهجين تحويل الموجة المتعدد (HMWT) حيث أن تحويل الموجة المتعدد المهجن هو خليط بين FFT و تحويل الموجة المتعدد المتعادل (MWT) كما أن تداخل التوافقي في نظام DMT بسبب انتشار دويل تم تقليله باستخدام تقنية الغاء التشخير المتعدد (PCC). إن نتائج المحاكاة نفذت على ثلاثة أنواع من قنوات الاتصال: قناة الإرسال ذات الضوضاء جاوسية (AWGN) وقناة الإرسال ذات تردد الأضعاف اللانهائي وقناة الإرسال ذات تردد الأضعاف الأدنى. أن منظومة PCC-DMT المستندة على تحويل الموجة المتعدد (MWT) حققت أداء أفضل من منظومة PCC-DMT المستندة على تحويل الموجة المتعدد (MWT) في كل أنواع القنوات.
1. Introduction

The major goal of modern communications is the development of a reliable high-speed wireless communication system that supports high user mobility. The next generation of wireless systems will require higher data quality than current cellular mobile radio systems and should provide higher bit rate services. In other words, the next generation of wireless systems are supposed to have better quality and coverage, be more powerful and bandwidth efficient, and deploy diverse environments [1]. The ever-increasing demand for wireless and multimedia applications such as video streaming keeps pushing future wireless local area network (WLAN) systems to support much higher data rates (100 MB/s up to 1 GB/s) at high link reliability and over greater distances. Next-generation wireless communication systems are focused on increasing the link throughput (bit rate), the network capacity, and the transmit range [2]. A multi-band orthogonal frequency division multiplexing (OFDM) ultra wideband system is being considered for the physical layer of the new IEEE wireless personal area network (WPAN) standard, IEEE 802.15.3a [3,4]. The standard is targeting high data transmission rates of 110 Mb/s over 10 m, 220 Mb/s over 4 m and 480 Mb/s over 1 m. The IEEE 802.15.3a transceivers will be used in portable devices, such as camcorders, and laptops, as well as in fixed devices, such as TVs and desktops. Therefore the link throughput (bit rate), the network capacity and bandwidth efficiency are very important issues to be addressed [5].

DMT is similar to OFDM, with the difference that DMT carries different numbers of bits on different subchannels. This signaling scheme leads to a better usage of the channel capacity [6]. DMT systems exploit the capabilities of orthogonal subcarriers to cope efficiently with narrowband interference, high frequency attenuations and multipath fading's with the help of simple equalization filters [7]. An important feature of DMT is the possibility to allocate the number of bits per subcarrier according to its corresponding signal-to-noise ratio (SNR), typically known as bit-loading [8]. To perform bit loading, the SNR of each sub-channel is estimated during the modem’s training phase. Then, based on the measured SNR, the appropriate bit loading is assigned to that channel to maximize the modem’s throughput [9].

The DMT system can be regarded as a filter bank in transmultiplexer configuration [10-12]. Typically, the filter banks used for this purpose are discrete Fourier transform (DFT)
filter banks which can be implemented efficiently with the FFT. The filters in these DFT filter banks provide poor separation between adjacent subchannels [13].

The block diagram of the DMT system is shown in Figure (1). The input data stream is grouped into quadrature amplitude modulation (QAM) sub-symbols. A complex-to-real (IFFT) used for modulation is performed to convert QAM subsymbols to real ones. Then the last samples of each real-valued data vector are copied and prefixed to the data vector then the parallel to serial converter (P/S) is done for this data. At the receiver, the channel outputs are converting from serial to parallel (S/P). After removing the samples corresponding to the cyclic prefix the FFT which acts as the demodulation operation [14, 15].

\[\text{Figure (1). Block diagram of the DMT system.}\]

Many researchers replaced the FFT with wavelet transform (WT) to improve the BER performance of the DMT system and this DMT is named discrete wavelet multitone (DWMT). The main difference between ordinary FFT and WT is the FFT uses the sine and cosine as a basis functions but the WT uses the wavelet function and scaling function as a basis functions [16, 17].

2. Multi-Wavelet Transform (MWT)

The Multiwavelet (MWT) uses more than one scaling function and wavelet function. Multiwavelets have some advantages in comparison to scalar ones (ordinary wavelets that have been mentioned). For example, such features as short support, orthogonality, symmetry, and vanishing moments are known to be important in signal processing. A scalar wavelet can not posses all these properties at the same time. On the other hand, a multiwavelet system can
have all of these properties simultaneously. This suggests that multiwavelets could perform better in various applications [18].

For notational convenience, the set of scaling functions can be written using the vector notation \( \Phi(t) = [\phi_1(t), \phi_2(t), ..., \phi_r(t)]' \), where \( \Phi(t) \) is called the multiscale function. Likewise, the multiwavelet function is defined from the set of wavelet functions as \( \Psi(t) = [\psi_1(t), \psi_2(t), ..., \psi_r(t)]' \). When \( r = 1 \), \( \Psi(t) \) is called a scalar wavelet, or simply wavelet. While in principle, \( r \) can be arbitrarily large, the multiwavelets up to date are primarily for \( r = 2 \) [19].

\[
\Phi(2^j t) = \sum_k H_{j+1}(k) \Phi(2^{j+1} t - k)
\]

\[
\Phi(2^j t) = \sum_k G_{j+1}(k) \Phi(2^{j+1} t - k)
\]

Note, however, that \( H_k \) and \( G_k \) are matrix filters, i.e. \( H_k \) and \( G_k \) are \( r \times r \) matrices for each one. The matrix elements in these filters provide more degrees of freedom than a traditional scalar wavelet. These extra degrees of freedom can be used to incorporate useful properties into the multiwavelet filters, as mentioned before. However, the multi-channel nature of multiwavelets also means that the subband structure resulting from passing a signal through a multifilter bank is different. The two-scale Eqs.(1) and (2) can be realized as a matrix filter bank (as shown in Figure (2)) operating on \( r \) input data streams and filtering them into \( 2^r \) output data streams, each of which is down-sampled by a factor of two.

Figure (2). Analysis and Synthesis of a single level DMWT.

In the scalar-valued expression \( V_{j,k}^{l} \), where \( j \) refers to the scale, \( k \) refers to the translation, and \( l \) refers to the sub-channel or vector row. It should be mentioned that the term discrete MWT (DMWT) is the same as MWT since the processing here in discrete form always.

The multiwavelet have many types and this work deals with a GHM-type constructed by Geronimo, Hardian and Massopust for their smoothing compared with other types of MWT. Where \( H_k \) for GHM system are four scaling matrices \( H_0, H_1, H_2, \) and \( H_3 \) [20]:

58
\[
H_0 = \begin{bmatrix}
\frac{3}{5\sqrt{2}} & \frac{4}{3} \\
\frac{1}{10\sqrt{2}} & -\frac{1}{20}
\end{bmatrix},
H_1 = \begin{bmatrix}
\frac{3}{5\sqrt{2}} & 0 \\
\frac{1}{10\sqrt{2}} & \frac{-3}{\sqrt{2}}
\end{bmatrix},
H_2 = \begin{bmatrix}
0 & \frac{3}{9} \\
\frac{-3}{20} & \frac{-3}{10\sqrt{2}}
\end{bmatrix},
H_3 = \begin{bmatrix}
0 & 0 \\
\frac{-1}{20} & 0
\end{bmatrix}
\]

and four wavelet matrices \( G_0, G_1, G_2, \) and \( G_3 \):
\[
G_0 = \begin{bmatrix}
\frac{1}{3} & \frac{-3}{10\sqrt{2}} \\
\frac{1}{10\sqrt{2}} & \frac{1}{10}
\end{bmatrix},
G_1 = \begin{bmatrix}
\frac{9}{20} & \frac{-1}{\sqrt{2}} \\
\frac{9}{10\sqrt{2}} & 0
\end{bmatrix},
G_2 = \begin{bmatrix}
\frac{9}{20} & \frac{-3}{10\sqrt{2}} \\
\frac{9}{10\sqrt{2}} & \frac{-3}{10}
\end{bmatrix},
G_3 = \begin{bmatrix}
\frac{1}{20} & 0 \\
\frac{1}{10\sqrt{2}} & 0
\end{bmatrix}
\]

2.1 Preprocessing

The low pass filter \( H \) and high pass filter \( G \) consist of coefficients corresponding to the dilation Eq.(1) and wavelet Eq.(2). But, in the multiwavelet setting these coefficients are \( r \) by \( r \) matrices, and during the convolution step they must multiply vectors (instead of scalars). This means that multifilter banks need \( r \) input rows. Here \( r = 2 \) and two data streams will be used as inputs to the multifilter. An Over-Sampling Scheme (repeated row) will be used because its convenient to the case of one-dimensional signal [19], where the input length 2 vectors are formed from the original signal as
\[
v_{\alpha,k} = \begin{bmatrix}
v_{0,k}^0 \\
v_{0,k}^1
\end{bmatrix} = \begin{bmatrix}
X_k \\
\alpha \cdot X_k
\end{bmatrix} \quad k = 0,1,\ldots,N-1
\]

For the GHM case \( \alpha = 1/\sqrt{2} \), where \( \alpha \) is the preprocessing factor.

3. Polynomial Cancellation Coding (PCC)

Polynomial cancellation coding (PCC) is a coding method in which the information to be transmitted is modulated onto weighted groups of subcarriers rather than onto individual subcarriers.

In Ref. [21] an OFDM system is designed with PCC where the high-speed data to be transmitted is divided into \( n \) lower speed parallel channels. The data in the \( k^{th} \) parallel channel in the \( i^{th} \) symbol period is represented by \( d_{k,i} \). This will in general be a complex value. The data values \( d_0,i \ldots d_{n-1,i} \) determine the values \( a_0,i \ldots a_{n-1,i} \) which modulate the \( N \) subcarriers in the \( i^{th} \) symbol period. For normal OFDM \( n = N \), and \( a_{k,i} = d_{k,i} \); one data value is used to modulate each subcarrier. With PCC, the data to be transmitted is mapped onto weighted groups of subcarriers. For example, to apply PCC to pairs of subcarriers, the subcarriers in
each pair must have relative weightings +1 and −1. In this case \( n = \frac{N}{2} \). The first data value in each symbol period is used to modulate the first two subcarriers: \( a_{0,i} = d_{0,i} \), \( a_{1,i} = -d_{0,i} \).

The demodulated subcarriers are then weighted and added to generate the data estimates \( v_{0,i} \ldots v_{n-1,i} \). For a pair of PCC-OFDM subcarriers \( z_{2M,i} \), \( z_{2M+1,i} \) an estimate is calculated using \( v_{M,i} (z_{2M,i} - z_{2M+1,i})/2 \) [22].

4. Proposed PCC-DMT system based on HMWT

The HMWT is a combination between FFT and MWT and therefore it has all the advantages of the FFT and MWT. Since the MWT is the same as WT but with two wavelet functions and two scaling functions and that leads to enhance the BER performance of DMT system in AWGN flat faded channel. In selective fading channel, the MWT gives worse BER performance in comparison to FFT since the BER is constant at certain signal to noise ratio (SNR) and gives a straight line but that does not happen in FFT and therefore the FFT will increase the orthogonality of MWT filters in HMWT.

The procedure steps to obtain on HMWT are:

Step 1: Apply the two dimensional FFT on each scaling matrix \( H \) and wavelet matrix \( G \) given in Eqs.(1) and (2) respectively. Assume \( Y \) any matrix (scaling or wavelet), then the two dimensional FFT for this matrix can be denoted by \( Z \) as:

\[
Z(k_1, k_2) = \sum_{r=1}^{N} \sum_{s=1}^{M} Y(r, s) e^{-\frac{2\pi i (r-1)(k_1-1)}{N}} e^{-\frac{2\pi i (s-1)(k_2-1)}{M}}
\]

(6)

where \( N \) is the number of elements in each row and \( M \) is the number of elements in each column (here \( N=M=2 \)) and \( k_1 = k_2 = 1, 2 \).

Step 2: Multiply the \( Z \) matrix which is calculated from step 1 with itself for two times by using corresponding elements multiplication method as shown in the following steps:

a) Change the dimensions of the \( Z \) matrix from \( 2*2 \) to \( 4*4 \) as shown:

\[
Z = \begin{bmatrix}
z_{11} & z_{12} \\
z_{21} & z_{22}
\end{bmatrix}
\rightarrow
Z = \begin{bmatrix}
z_{11} & 0 & 0 & 0 \\
0 & z_{12} & 0 & 0 \\
0 & 0 & z_{21} & 0 \\
0 & 0 & 0 & z_{22}
\end{bmatrix}
\]

(7)

b) Multiply the \( Z \) matrix in Eq.(7) by itself for two times, and assume \( F \) is the result of multiplication, then:

\[
F = Z * Z * Z
\]
\[
F = \begin{bmatrix}
  z_{11}^3 & 0 & 0 & 0 \\
  0 & z_{12}^3 & 0 & 0 \\
  0 & 0 & z_{21}^3 & 0 \\
  0 & 0 & 0 & z_{22}^3
\end{bmatrix}
\]

(8)

c) Change the dimensions of the \( F \) matrix in Eq.(8) from \( 4 \times 4 \) to \( 2 \times 2 \) as shown:

\[
F = \begin{bmatrix}
  f_{11} & 0 & 0 & 0 \\
  0 & f_{12} & 0 & 0 \\
  0 & 0 & f_{21} & 0 \\
  0 & 0 & 0 & f_{22}
\end{bmatrix} \quad \rightarrow \quad F = \begin{bmatrix}
  f_{11} & f_{12} \\
  f_{21} & f_{22}
\end{bmatrix}
\]

(9)

Where \( f_{ij} = z_{ij}^3 \) for \( i=j=1, 2 \).

Step 3: Apply the two dimensional IFFT on the matrix \( F \) in Eq.(9). Assume \( \tilde{F} \) is the two dimensional IFFT for matrix \( F \) and calculate using Eq.(10):

\[
\tilde{F} (r, s) = \frac{1}{NM} \sum_{k_1=1}^{N} \sum_{k_2=1}^{M} F (k_1, k_2) e^{\frac{2\pi i}{N}(r-1)(k_1-1)} e^{\frac{2\pi i}{M}(s-1)(k_2-1)}
\]

(10)

Applying the steps 1-3 on all scaling matrices \( H \) and wavelet matrices \( G \) to obtain new GHM multifilter bank. Assuming the hybrid four scaling matrices that corresponding to GHM system are \( A_0, A_1, A_2, \) and \( A_3 \) then:

\[
A_0 = \begin{bmatrix}
  1.0023 & 1.0850 \\
  -0.5619 & -0.6348
\end{bmatrix}, \quad A_1 = \begin{bmatrix}
  0.9705 & 0.8100 \\
  1.0091 & 1.1650
\end{bmatrix},
\]

(11)

\[
A_2 = \begin{bmatrix}
  0 & 0 \\
  0.1519 & -0.1384
\end{bmatrix}, \quad A_3 = \begin{bmatrix}
  0 & 0 \\
  -0.1250 \times 10^{-3} & 0
\end{bmatrix},
\]

and hybrid four wavelet matrices \( B_0, B_1, B_2, \) and \( B_3 \)

\[
B_0 = \begin{bmatrix}
  -0.0481 & -0.0780 \\
  0.0486 & 0.0788
\end{bmatrix}, \quad B_1 = \begin{bmatrix}
  1.3129 & -1.6423 \\
  -1.5989 & 1.2150
\end{bmatrix},
\]

(12)

\[
B_2 = \begin{bmatrix}
  1.0631 & -0.9689 \\
  1.0739 & -0.9788
\end{bmatrix}, \quad B_3 = \begin{bmatrix}
  -0.8750 \times 10^{-3} & 0 \\
  -0.8839 \times 10^{-3} & 0
\end{bmatrix},
\]

As in MWT the coefficients are \( r \) by \( r \) matrices, and during the convolution step they must multiply vectors (instead of scalars). This means that hybrid multifilter banks need two data streams and therefore a repeated row will be used. Because of the repeated row is formed input length 2 vectors from the original signal by dividing on \( \alpha ; \alpha \) is taken equal to -1 in the proposed system to perform the repeated row and PCC in the same time.
5. Simulation results

Matlab 2009a is used to compute the BER performance of the proposed system. The number of subcarriers taken (the number of FFT points) equals to 64. Three types of channels are used: The AWGN channel, AWGN with Raleigh flat fading channel and AWGN with Raleigh selective fading channel. The results show that the BER performance for proposed system in AWGN with Raleigh flat fading channel for different Doppler frequency shift (5 and 500) Hz. Also the results show that the BER performance for proposed system in AWGN with Raleigh selective fading channel for different Doppler frequencies shift (5, 100 and 500) Hz, path delay (1, 2 and 8) samples and attenuation of the second path gain (-8 and -11) dB. In all schemes, the uniform bit-loading across the subchannels with number of bits (b) = 64.

The BER performance for proposed system compared with different DMT systems based on FFT, MWT and HMWT. In DMT system that based on MWT or HMWT, \( \alpha = \frac{1}{\sqrt{2}} \) but in PCC-DMT system that based on FFT, MWT or HMWT, \( \alpha = -1 \).

5.1 Performance of the proposed system in AWGN channel

As shown in Figure (3) the gain at BER = 10^{-3} is about 4.145 dB when using DMT system based on HMWT as compared with the DMT system based on MWT. The gain of DMT system based on HMWT is 22.584 dB as compared with the DMT system based on FFT. The gain for PCC-DMT system is about 0.5 over the DMT system without PCC and for all transforms.

5.2 Performance of the proposed system in flat fading channel

Figure(4) shows the BER performance for the proposed system in AWGN with Raleigh flat fading channel at Doppler frequency shift = 5 Hz. At BER = 10^{-3}, the gain is about 4.477 dB and 22.41 dB when using DMT based on HMWT system as compared with the DMT system based on MWT and the DMT system based on FFT respectively. The performance of proposed system is nearly the same performance of DMT system based on HMWT but it given gain about 3.683 dB and 21.81 dB as compared with itself when it's based on MWT and FFT respectively.

Figure(5) shows the BER performance for the proposed system in AWGN with Raleigh flat fading channel at Doppler frequency shift = 500 Hz. The gain between all
systems in this Figure is nearly the same gain that obtained in Figure(4). but the BER performance in Figure(4) is better than the BER performance in Figure(4). with gain about 4 dB for all systems.

5.3 Performance of the proposed system in selective fading channel

Figures(6) and (7) show the BER performance for the proposed system in AWGN with Raleigh selective fading channel at Doppler frequency shift = 5 Hz, path delay = 1 and 2 samples respectively and attenuation of the second path gain = -11 dB. In Figure(6), at BER = $10^{-3}$, the gain is about 3.938 dB and 22.298 dB when using DMT based on HMWT system as compared with the DMT system based on MWT and the DMT system based on FFT respectively. The performance of PCC-DMT system based on HMWT is nearly the same performance of DMT system based on HMWT but the first system given gain about 3.566 dB and 22.01 dB as compared with itself but when it is based on MWT and FFT respectively. The gain between all systems in Figure(7). is nearly the same gain that obtained in Figure(6). but the BER performance in Figure(6). is better than the BER performance in Figure(7). because the path delay is increased from 1 sample to 2 samples.

Figures(9) and (10) show the BER performance for the proposed system in AWGN with Raleigh selective fading channel at Doppler frequency shift = 500 Hz, path delay = 1 and 2 samples respectively and attenuation of the second path gain = -11 dB. The gain between all systems in Figures(9) and (10) is nearly the same gain that obtained in Figures(6) and (7) but the BER performance in Figures (9) and (10) is increased due to increase the Doppler frequency shift to 500 Hz.

Figures (8) and (11) show the BER performance for the proposed system in AWGN with Raleigh selective fading channel at different Doppler frequency shift = 5 Hz and 500 Hz respectively, path delay = 8 samples and attenuation of the second path gain = -8 dB. The gain between all systems in these figures is nearly the same gain that obtained in Figures(6) and (9) respectively but the BER performance in Figures(8) and (11) is not better than the BER performance in Figures (6) and (9) respectively because the attenuation of the second path gain is increased and that lead to decrease the BER performance.
6. Conclusions

In this paper a proposed transform HMWT is used based on both FFT and MWT. The reason of using the FFT for GHM multiwavelet filter banks is to increase the orthogonality of these filters where the orthogonality will increase on the rows and columns and that leads to more orthogonal filters. FFT were taken for each GHM filter bank then multiply the result with itself for three times to obtain best results as in wavelet transform when taken more level of decomposition that gives best results. The comparison between the BER performance for a proposed system and other systems is taken for three different types of channels. Simulation results of PCC-DMT system based on HMWT show a good SNR gain improvement compared with DMT system with and without PCC and either based on FFT or MWT.

Many techniques can be used as future work to increase system performance such as channel coding, channel equalization and using multi FFT.

![Figure (3). Performance of different types of multi-carrier system under AWGN channel.](image-url)
Figure (4). Performance of different types of multi-carrier system under AWGN channel with flat fading channel at Doppler frequency shift = 5 Hz.

Figure (5). Performance of different types of multi-carrier system under AWGN channel with flat fading channel at Doppler frequency shift = 500 Hz.
Figure (6). Performance of different types of multi-carrier system under AWGN channel with selective fading channel at Doppler frequency shift = 5 Hz, path delay = 1 sample and attenuation of the second path gain = -11 dB.

Figure (7). Performance of different types of multi-carrier system under AWGN channel with selective fading channel at Doppler frequency shift = 5 Hz, path delay = 2 samples and attenuation of the second path gain = -11 dB.
Figure (8). Performance of different types of multi-carrier system under AWGN channel with selective fading channel at Doppler frequency shift = 5 Hz, path delay = 8 samples and attenuation of the second path gain = -8 dB.

Figure (9). Performance of different types of multi-carrier system under AWGN channel with selective fading channel at Doppler frequency shift =500 Hz, path delay = 1 sample and attenuation of the second path gain = -11 dB.
Figure (10). Performance of different types of multi-carrier system under AWGN channel with selective fading channel at Doppler frequency shift =500 Hz, path delay = 2 samples and attenuation of the second path gain = -11 dB.

Figure (11). Performance of different types of multi-carrier system under AWGN channel with selective fading channel at Doppler frequency shift =500 Hz, path delay = 8 samples and attenuation of the second path gain = -8 dB.
7. References


