Kinematic Analysis of Bicycle Pedaling

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Abstract

In this study, the kinematic equations are derived to describe the angles, velocities, and accelerations of the leg of bike rider when changing the height and angle of seat tube. These equations are useful to get the optimum seat position, (in separated study), for best performance and efficiency.

The pedaling motion model was simulated as four-bar mechanism (the foot was not considered), and by using polar form of complex number notation to get the kinematic equations. These equations are agreed with MATLAB/Simulink/SimMechanics/Four-Bar Model, as well as, the graphical method. The derived equations (kinematic equations) can describe the crossed and open four-bar mechanism.

Key Words: Bike Fit, Pedaling , Kinematic Equations, Four-Bar Mechanism.

المستخلص

في هذا البحث تم اشتقاق معادلات الزوايا و السرع والتعجيل لحركة ساق راكب الدراجة نسبة إلى حركة الدواسة في حالة تغيير ارتفاع وزاوية المقعد, وذلك لإيجاد, (في بحث منفصل), الموقع المثالي للمقعد للحصول على أفضل أداء واقل جهد على السائق. ومثلت حركة الساق مع الدواسة بميكانيكية التراكيب الأربعة(four-bar mechanism) حيث تم إهمال دور القدم.

باستخدام المحاور القطبية للإعداد المركبة تم الحصول على المعادلات الحركية حيث قورنت وبنجاح مع برنامج المحاكاة المتوفر في برنامج الماتلاب (MATLAB/SIMULINK), إضافة إلى طريقة الرسم التقليدية (Graphical Method). والمعادلات الحركية الكيناماتيكية تصف ميكانيكية التراكيب الأربعة للتراكيب المتقاطعة والتراكيب المفتوحة (Crossed and Open four-bar mechanism).

1. Introduction

Correct body position on the bike (i.e. equipment configuration, seat tube angle, correct seat height and the foot placement on the pedal) are all interrelated and are collectively known as the bike fit. For this reason, the bike fit has the outcome of proper muscle recruitment, achievable pedaling rate, power and ultimately maximum pedaling efficiency[1]. Where the pedaling efficiency is the difference between the human power expanded and the actual power delivered to the road. The maximum efficiency can be attained through training of proper pedaling technique and also through a proper bike fit to maximize joint angles of individual [2].

The bike fit adjusts the angles of the 'lever' of the hip, knee and ankle joints as they relate to the foot-to-pedal interface to achieve the overall performance objective. An improper bike bit will result in reduced efficiency through less than optimal muscle recruitment (improper angles of the levers and potentially, in serious cases, injuries could result from an improper bike fit). Finally, the bike fit must always begin with the individual assessment (i.e. goals, fitness level, biking experience, flexibility, body measurement) then adjust/fit bicycle to the individual, being sure not to make the common mistake of fitting the individual to the bicycle[3]. Following is a list of bike fit adjustment/ variables [4]:

1- Seat tube angle-correct hip lever angle.
2- Seat height-achieve optimal performance height and angle of hip lever.
3- Seat position fore/aft-get knee directly over pedal and correct knee lever angle.
4- Adjustment of shoe cleats-correct knee and ankle lever angle.
5- Handle bar height, reach and size-maximize hip flexor/extender angle. Correct back and shoulder positioning for comfort and aerodynamics.
6- Foot placement on pedal-correct ankle lever angle at the man/machine interface.
7- Crank arm length- needs to be optimum length for cycling purpose.
8- Power meter technology to assess bike fit- technology of power meters uses biomechanical technology to assess and improve power and thus performance.

The motion of cycling as if it were the face of clock, beginning at approximately the 11o'clock position[3]:

1- Preparatory phase: 11:00 to 1:00,
2- Power phase: 1:00 to 5:00,
3- Follow through phase: 5:00 to7:00, and
4- Recovery phase: 7:00 to 11:00.

The Preparatory Phase: prepares the leg and foot for the power phase. This phase begins with the knee in its most flexed position, ready for the push through into the movement phase. The ankle will tend to go from a slightly plantar flexed position at 11:00 to a nearly neutral at the 1:00 position.

The Power Phase: during the power phase, hip and knee extension continue, the most effective force applied is that one which is perpendicular to the crank clearly, the 3:00 position is the peak of the movement phase(when the seat position above the pedal axis). During this phase, particularly in the lower part of the movement phase knee extension is also occurring.

The Follow Through Phase: as the pedaling foot is moving from 5:00 to 7:00 in follow through, both feet are in the least effective position to produce power . The joint action here is knee flexion, with some slight hip extension and flexion as the foot moves through the bottom of the circle and begins upward travel and the ankle joint will begin a slight plantar-flexion, helping to keep applied force as nearly tangential as possible.

Recovery: the primary joint movement at the hip is flexion. Some of the energy expended by the leg in the power phase will be wasted and used to help push the recovering leg up to the top of its phase, where knee is flexion and the ankle joint will continue to be slightly plantar-flexed beginning to move slightly toward neutral. A viewing the motion of pedaling a bicycle can be simplified by considering only one leg at a time while the opposite leg will be doing the exact same motions, only in 180 degree opposition. If the right knee is flexing at any given moment, the left knee will be extending at the same moment. Thus the motion of only one leg at a time will be considered. The most pedal effective, force applied is that which is perpendicular to the crank. The ankle joint will ideally remain nearly neutral (pedal level), so that all force applied through power phase will follow a tangential line, perpendicular to the crank [1].

2. Literature review

There are many researchers who have studied the subjects which related to bicycle as: Jim M. Papadopoulos [5], studied the bicycle pedaling and the forces which exerted on the muscles by using two actuators for modeling the real muscles one to move the thigh and another to move the leg, by used real model as four-bar mechanisms.

Jim M. Papadopoulos, R. Scott Hand and Andy Ruina [6], presented the linearized equations of motion for lateral motion for a basic bicycle (not for pedaling).

D. G. Kooijman, J.P.Meijaurd &A.L. Schwab [7], were Studied the bicycle stability by experimental test by equipped with sensor, data acquisition unit and laptop on the rear rack to measure forward speed, steering angle, lean rate and yaw rate, where the experimental results were within the results obtained from the linearized analysis on the simple bicycle model.

Grant Bullock, Davon Cabraloff, Jessica Hickman, Mark Mico, Laura Netcher & Dan Ward [3] discussed the muscle activity during phases of pedaling. They divided the pedal stroke into four individual phases of moment and identified the mechanical purpose of each phase.

3. The goal of this study

The main goal of this study is to derive kinematic equations of pedaling, which will use to get the optimum seat position. These equations will describe the positions (angles), the velocities (angular and linear velocities) and acceleration (tangential, radial and linear accelerations) of the thigh, calf, and crank, when changing angle and height of seat position.

4. Kinematic analysis for peddling

4.1 Vector position analysis

The vectors loop closes on itself making the sum of the vectors around the loop is zero [8]:

$R_{AO2}+R_{BA}-R_{BO4}-R_{O4O2}=Zero$

Substitute the complex number notation for each position vector gives:

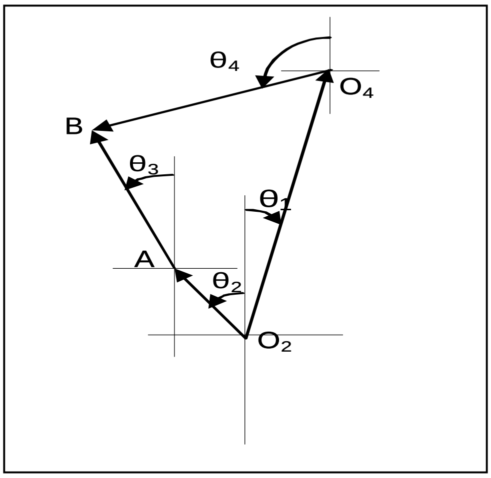


Figure (1). Modeling of pedaling as four-bar linkage.

$$ae^{j\theta_2} + be^{j\theta_3} - ce^{j\theta_4} - de^{-j\theta_1} = Zero \qquad (1)$$

where : a, b, c, and d are the length of pedaling crank, calf, thigh and ground link respectively.

θ_2, θ_3, θ_4, and θ_1 are the angles of pedaling crank, Calf, thigh and ground link respectively as shown in figure (1).

$\overline{O2O4}$: is the ground link (is the link between center of pedal to seat position).

$\overline{AO2}$: is pedal crank link.

$\overline{BA}$: is the calf link.

$\overline{BO4}$: is the thigh link.

θ_2 : is independent variable.

∵Euler identity: $e^{\pm j\theta} = (\cos\theta \pm j\sin\theta)$.

Substitute Euler identity in equation (1) gives:

$$a(\cos\theta_2 + j\sin\theta_2) + b(\cos\theta_3 + j\sin\theta_3) - c(\cos\theta_4 + j\sin\theta_4) - d(\cos\theta_1 - j\sin\theta_1) = Zero \qquad (2)$$

equation (2) can be separated into its real and imaginary parts and each set to zero:

real part:

$$a\cos\theta_2 + b\cos\theta_3 - c\cos\theta_4 - d\cos\theta_1 = Zero \qquad (3)$$

imaginary part:

$$a\sin\theta_2 + b\sin\theta_3 - c\sin\theta_4 + d\sin\theta_1 = Zero \qquad (4)$$

Square both sides of the real and imaginary equations and add them, then arrange, gives:

$$(\cos\theta_2\cos\theta_4 + \sin\theta_2\sin\theta_4) = [K_3 - K_2 * M * \cos\theta_2 + K_2 * N * \sin\theta_2 + K_1 * M * \cos\theta_4 - K_1 * N * \sin\theta_4] \qquad (5)$$

Where:

$$M = \cos\theta_1, \quad N = \sin\theta_1, \quad K_1 = \frac{d}{a}, \quad K_2 = \frac{d}{c}, \quad K_3 = \frac{a^2 - b^2 + c^2 + d^2}{2ac}$$

$$\because \sin\theta_4 = \frac{2\tan(\frac{\theta_4}{2})}{1 + (\tan(\frac{\theta_4}{2}))^2} \qquad (6)$$

$$\text{and } \cos\theta_4 = \frac{1 - (\tan(\frac{\theta_4}{2}))^2}{1 + (\tan(\frac{\theta_4}{2}))^2} \qquad (7)$$

Substituting equations (6) and (7) in equation (5) gives:

$$\tan^2\left(\frac{\theta_4}{2}\right)[K_3 - K_2 * M * \cos\theta_2 + K_2 * N * \sin\theta_2 + \cos\theta_2 - K_1 * M] + \tan\left(\frac{\theta_4}{2}\right)[2\sin\theta_2 + 2K_1 * N] + [K_3 - K_2 * M * \cos\theta_2 + K_2 * N * \sin\theta_2 - \cos\theta_2 + K_1 * M] = Zero$$

The solution of quadratic is:

$$\theta_4 = 2 * tan^{-1}\left[\frac{-(B) \pm \sqrt{(B^2 - 4 * A * C)}}{2 * A}\right] \quad (8)$$

Where:

A=$[K_3 - K_2 * M * cos\,\theta_2 + K_2 * N * sin\,\theta_2 + cos\theta_2 - K_1 * M]$

B=$-[2Sin\theta_2 + 2K_1 * N]$

C=$[K_3 - K_2 * M * cos\,\theta_2 + K_2 * N * sin\,\theta_2 - cos\theta_2 + K_1 * M]$

The solution for angle θ_3 is essentially similar to that for θ_4:

Rearranging equations (3) and (4) as follows:

$c\ cos\ \theta_4 = (a\ cos\ \theta_2 + b\ cos\ \theta_3 - d\ cos\ \theta_1)$ (9)

$c\ sin\ \theta_4 = (a\ sin\ \theta_2 + b\ sin\ \theta_3 + d\ sin\ \theta_1)$ (10)

Square both sides of equations (9) and(10) respectively, and add them, then arrange and substitute the half angle identities will convert the sin θ_3 and cos θ_3 terms to tan θ_3, gives:

$$tan^2\left(\frac{\theta_3}{2}\right)[cos\,\theta_2(1 + K_4 * M) - K_4 * N * sin\,\theta_2 + K_5 - K_1 * M] + tan\left(\frac{\theta_3}{2}\right)[2K_1 * N + 2sin\theta_2] + [cos\theta_2(K_4 * M - 1) - K_4 * N * sin\,\theta_2 + K_5 + K_1 * M] = Zero$$

Where:

$$K_4 = \frac{d}{b}, \qquad K_5 = \frac{-a^2 - b^2 + c^2 - d^2}{2ab}$$

The solution of quadratic is:

$$\theta_3 = 2 * tan^{-1}\left[\frac{-(E) \pm \sqrt{(E^2 - 4 * D * F)}}{2 * D}\right] \quad (11)$$

Where:

D=$[cos\,\theta_2(1 + K_4 * M) - K_4 * N * sin\,\theta_2 + K_5 - K_1 * M]$

E=$-[2K_1 * N + 2Sin\theta_2]$

F= $[cos\theta_2(K_4 * M - 1) - K_4 * N * sin\,\theta_2 + K_5 + K_1 * M]$

4.2 Vector velocity analysis

For velocity expression differentiate the equation (1) respect to time, gives:

$j\frac{d\theta_2}{dt}ae^{j\theta_2} + j\frac{d\theta_3}{dt}be^{j\theta_3} - j\frac{d\theta_4}{dt}ce^{j\theta_4} + j\frac{d\theta_1}{dt}de^{-j\theta_1}$=Zero

$j\omega_2 ae^{j\theta_2} + j\omega_3 be^{j\theta_3} - j\omega_4 ce^{j\theta_4}$=Zero (12)

Where : $\frac{d\theta_1}{dt} = zero$ (because the linkage is stationary).

And let $\frac{d\theta_2}{dt} = \omega_2$, $\frac{d\theta_3}{dt} = \omega_3$, and $\frac{d\theta_4}{dt} = \omega_4$

Where the θ_3 and θ_4 are obtained from position analysis equation (11) and (8) respectively.

Substitute the Euler identity in equation (12), gives:

$$j\omega_2 a(cos\theta_2 + jsin\theta_2) + j\omega_3 b(cos\theta_3 + jsin\theta_3) - j\omega_4 c(cos\theta_4 + jsin\theta_4)=\text{Zero} \quad (13)$$

Separate equation (13) into its real and imaginary parts and set each part to zero gives:

real part:

$$-a\omega_2\ Sin\theta_2 - b\omega_3 Sin\ \theta_3 + c\omega_4\ Sin\ \theta_4=Zero \quad (14)$$

imaginary part:

$$a\omega_2\ Cos\theta_2 + b\omega_3 Cos\ \theta_3 - c\omega_4 Cos\ \theta_4=Zero \quad (15)$$

solving equations(14) and(15) gives:

$$\omega_3 = \frac{a\omega_2}{b}\frac{\sin(\theta_4 - \theta_2)}{\sin(\theta_3 - \theta_4)} \quad (16)$$

$$\omega_4 = \frac{a\omega_2}{c}\frac{\sin(\theta_2 - \theta_3)}{\sin(\theta_4 - \theta_3)} \quad (17)$$

∵The relative velocity:

$$V_A + V_{BA} - V_B = zero$$

Where:

$$V_A = j\omega_2 a e^{j\theta_2} = a\omega_2(-\sin\theta_2 + j\cos\theta_2) \quad (18)$$

$$V_{BA} = j\omega_3 b e^{j\theta_3} = b\omega_3(-sin\ \theta_3 + j\ cos\ \theta_3) \quad (19)$$

$$V_B = j\omega_4 c e^{j\theta_4} = c\omega_4(-sin\ \theta_4 + j\ cos\ \theta_4) \quad (20)$$

4.3 Vector acceleration analysis

Now, differentiating equation(12) versus time to obtain an expression for acceleration in the linkage gives:

$$(j^2 a\omega_2^2 e^{j\theta_2} + ja\alpha_2 e^{j\theta_2}) + (j^2 b\omega_3^2 e^{j\theta_3} + jb\alpha_3 e^{j\theta_3}) - (j^2 c\omega_4^2 e^{j\theta_4} + jc\alpha_4 e^{j\theta_4}) = \text{zero}$$

Simplifying and grouping terms give:

$$(ja\alpha_2 e^{j\theta_2} - a\omega_2^2 e^{j\theta_2}) + (jb\alpha_3 e^{j\theta_3} - b\omega_3^2 e^{j\theta_3}) - (jc\alpha_4 e^{j\theta_4} - c\omega_4^2 e^{j\theta_4}) = \text{zero} \quad (21)$$

Where:

α_2 = angular acceleration of pedal = $\frac{d\omega_2}{dt}$ (rad/sec).

α_3 = angular acceleration of calf = $\frac{d\omega_3}{dt}$ (rad/sec).

α_4 = angular acceleration of thigh= $\frac{d\omega_4}{dt}$ (rad/sec).

Substituting the Euler identity in each term of equation (21) then collecting all real and all imaginary terms separately gives:

Real part:

$$-a\alpha_2 \sin\theta_2 - a\omega_2^2 \cos\theta_2 - b\alpha_3 \sin\theta_3 - b\omega_3^2 \cos\theta_3 + c\alpha_4 \sin\theta_4 - c\omega_4^2 \cos\theta_4 = zero \qquad (22)$$

Imaginary part:

$$a\alpha_2\, Cos\theta_2 - a\omega_2^2\, Sin\,\theta_2 + b\alpha_3\, Cos\,\theta_3 - b\omega_3^2\, Sin\theta_3 \; - c\alpha_4\, Cos\,\theta_4 + c\omega_4^2\, Sin\,\theta_4 = Zero \qquad (23)$$

Solving equations (22) and (23) simultaneously gives:

$$\alpha_3 = \frac{R*S - P*U}{P*T - Q*S} \qquad (24)$$

$$\alpha_4 = \frac{R*T - Q*U}{P*T - Q*S} \qquad (25)$$

Where:

$P = c\,sin\,\theta_4$

$Q = b sin\theta_3$

$R = a\alpha_2\, sin\,\theta_2 + a\omega_2^2\, cos\,\theta_2 + b\,\omega_3^2\, cos\,\theta_3 - c\,\omega_4^2\, cos\,\theta_4$

$S = c\,cos\,\theta_4$

$T = b\,cos\,\theta_3$

$U = a\alpha_2\, cos\,\theta_2 - a\omega_2^2\, sin\,\theta_2 - b\omega_3^2\, sin\,\theta_3 + c\omega_4^2\, sin\,\theta_4$

The relative acceleration and linear accelerations are:

$A_A + A_{BA} - A_B = zero$

$$\begin{aligned} A_A &= (A_A^t + A_A^n) = ja\alpha_2 e^{j\theta_2} - a\omega_2^2 e^{j\theta_2} \\ &= a\,\alpha_2(-\sin\theta_2 + j\cos\theta_2) - a\omega_2^2(\cos\theta_2 + j\sin\theta_2) \end{aligned} \qquad (26)$$

$$\begin{aligned} A_B &= (A_B^t + A_B^n) = jc\alpha_4 e^{j\theta_4} - c\omega_4^2 e^{j\theta_4} \\ &= b\alpha_3(-\sin\theta_3 + j\cos\theta_3) - b\omega_3^2(\cos\theta_3 + j\sin\theta_3) \end{aligned} \qquad (27)$$

$$\begin{aligned} A_{BA} &= (A_{BA}^t + A_{BA}^n) \\ &= jb\alpha_3 e^{j\theta_3} - b\omega_3^2 e^{j\theta_3} c\alpha_4(-\sin\theta_4 + j\cos\theta_4) \\ &\quad - c\omega_4^2(\cos\theta_4 + j\sin\theta_4) \end{aligned} \qquad (28)$$

Equations (24) to (28) are complete solution for the angular acceleration of the links and the linear acceleration of the joints in the pin jointed four bar linkage.

5. Results and discussion

To check the kinematic equations (derived equations) of four-bar mechanisms, an the example of four bar mechanism modeling was used as shown in figure (2) in MATLAB Package / Simulink / SimMechanics as shown in Table (1).

Table (1). Specifications of four-bar mechanisms.

Link	Length (cm)	Angle (degree)	Note: Consider as
Crank link(Shortest link)	12	60	Pedal of bike
Stationary link(Longest link)	86.7	0.0	Height of seat
Rocker link	60	83.96	Thigh of rider
Coupler link	100	29.52	Calf of rider

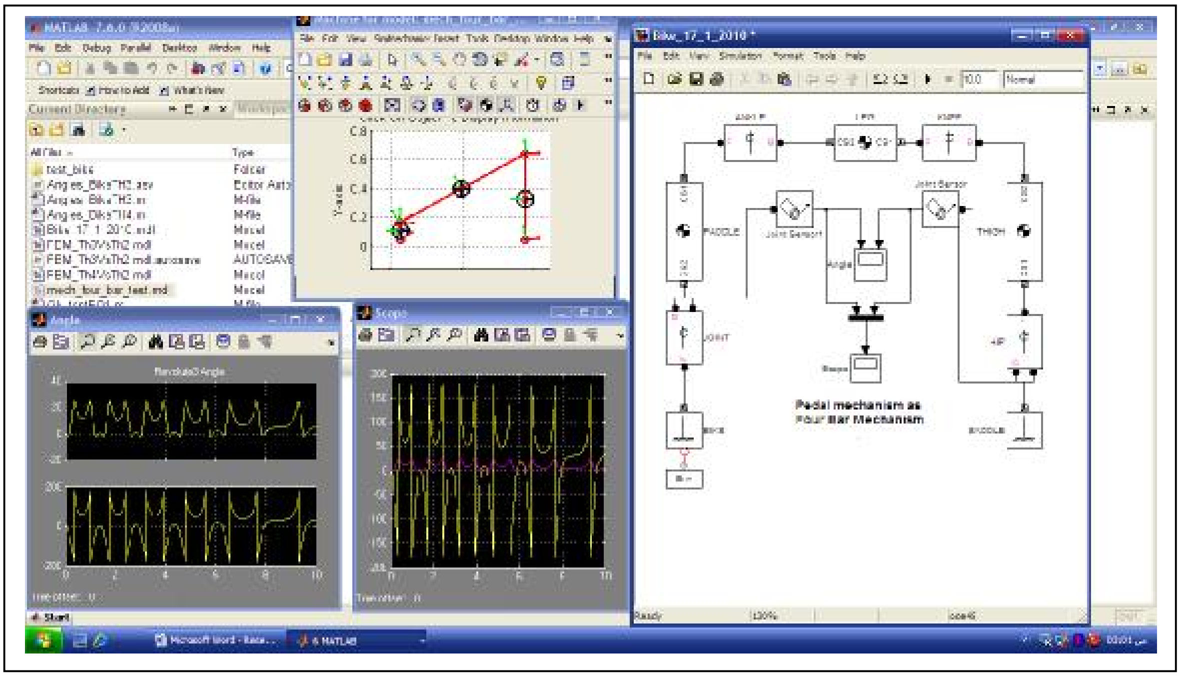
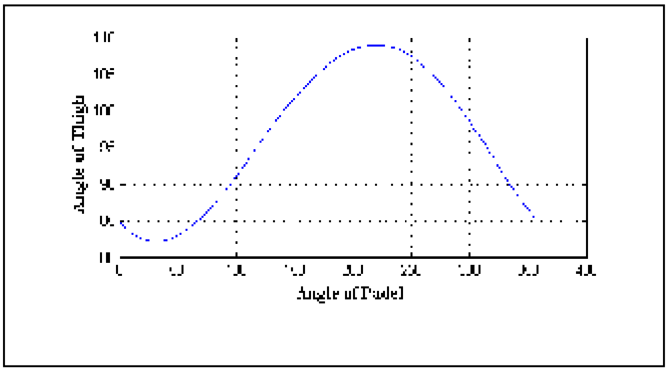
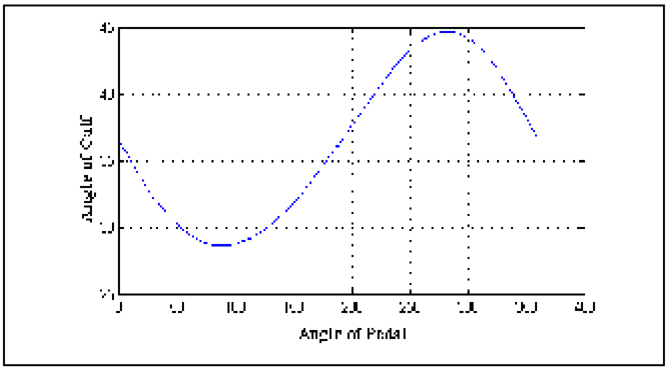


Figure (2). Simulation of four-bar mechanism in MATLAB/Simulink/Sim-mechanics.

By using MATLAB editor to calculate equations (8) and (11), the plot of the angle of thigh versus the pedal angle, and the angle of calf versus the pedal angle respectively as shown in figure (3), were coincided with the angles of thigh and calf versus pedal angle which obtained from four-bar modeling, (where the absolute angle of revoulte motion is mapped in angle or scope window(as shown in figure-2) on to the interval -180°, +180°, thus the angles are modified to be suitable for global coordinates).



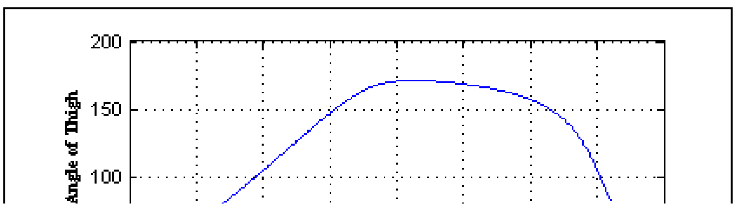


Figure(3). The angles of thigh and calf versus angle of pedal, from derived equations and four-bar model .

When the derived equations compared with graphical method and with the information of example of quadric cycle chain [8], the dimensions of four bar chain are given in Table (2). The derived kinematic equations coincide with result of the example as shown in figure (4), also .

Table (2). Dimensions of four-bar chain.

Link	Length (cm)	Angle (degree)	Angular velocity(rad/s)	Consider as
Crank link	15	60	10	Pedal of bike
Stationary link	22	0.0	0.0	Height of seat
Rocker link	18	73.76	7.0	Thigh of rider
Coupler link	20	12.39	-2.035	Calf of rider



Figure(4). Angular velocity of thigh and calf versus angle of pedal.

6. Conclusions

The derived equations in this study can be considered as equations to describe the kinematic motion of the four-bar mechanism.

The equation (8) has two solutions, obtained from the ± conditions on the radical. These two solutions as with any quadratic equation, may be of three types: real and equal, real and unequal, and complex conjugate. The complex conjugate solution means that the link lengths chosen are not capable of connection for the chosen value of the input angle θ_2. Thus the links must satisfy the Grashof Condition which satisfies crank-rocker :(length of shortest link(pedal length) + length of longest link(saddle position) < length of one remaining link (Thigh length) + length of other remaining link(calf length)).[8]

The real and unequal solution means that there are two values of θ_4 corresponding to any value of θ_2, these are referred to as the crossed and open configurations of the linkage, as the two circuits of the linkage.

Where the minus solution gives θ_4 for the open configuration (which was taken in this study), and positive solution gives θ_4 for the crossed configuration. Also, the minus solution gives θ_3 for the open configuration (which was taken in this study), and positive solution gives θ_3 for the crossed configuration.

7. References

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8. Nomenclature

$\overline{AO2}$	Vector of pedal link.
$\overline{BA}$	Vector of calf link.
$\overline{BO4}$	Vector of thigh link.
$\overline{O2O4}$	Vector of ground link (is the link between center of pedal to seat position).
ω_2	Angular velocity of pedal link (rad/sec).
ω_3	Angular velocity of calf link (rad/sec).
ω_4	Angular velocity of thigh link (rad/sec).
A_A^n	Normal acceleration of pedal link (m/sec^2).
A_A^t	Tangential acceleration of pedal link (m/sec^2).
A_{BA}^n:	Normal acceleration of calf link relative to pedal link (m/sec^2).
A_{BA}^t	Tangential acceleration of calf link relative to pedal link (m/sec^2).
A_B^n:	Normal acceleration of thigh link (m/sec^2).

A_B^t	Tangential acceleration of thigh link (m/sec^2).
V_A	Linear velocity of pedal link (m/sec).
V_B	Linear velocity of thigh link (m/sec).
V_{BA}	Linear velocity of calf link relative to pedal link (m/sec).
α_2	Angular acceleration of pedal link (rad/sec^2).
α_3	Angular acceleration of calf link (rad/sec^2).
α_4	Angular acceleration of thigh link (rad/sec^2).
a	Length of pedaling link (m).
b	Length of calf link (m).
c	Length of thigh link (m).
d	Length of seat tube link (m).
θ_1	Angle of seat tube link (degree).
θ_2	Angle of pedaling link (degree).
θ_3	Angle of calf link (degree).
θ_4	Angle of thigh link (degree).