Free Vibration of Axisymmetric Thin Oblate Shells Containing Fluid

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Abstract

A theoretical analysis for the free , axisymmetric, vibrations of an isotropic thin oblate spheroid shell filled partially or completely with an incompressible, non-viscous, irrotational fluid is considered. The Rayleigh – Ritz's method is used to obtain an approximate solution which coincides with the exact solution for the cases of an empty or completely filled shell.

The vibration of the shell is examined using the non – shallow shell theory. The analysis is based on considering the oblate spheroid as a continuous system constructed from two spherical shell elements matched at the continuous boundaries. Solutions are presented to show the effect of the angle of filling fluid on the shell natural frequencies. The effect of shell geometric parameters on the frequencies is also investigated. Natural frequencies are calculated for the shell in both empty and filled cases. It was found that their frequencies are decreased with the increase of fluid level in the shell. The analytical solution is compared with available test results. Good agreement is shown between test results found in the literature and predicted natural frequencies.

Keywords: Oblate spheroid, thin shells, axisymmetric spheroid, incompressible fluid.

الاهتزازات الحرة للقشريات المتناظرة المحور نحيفة الجدران شبه البيضوية الشكل المحتوية على مائع

المستخلص

يتناول هذا البحث تحليلاً نظرياً للاهتزازات الحرة للقشريات نحيفة الجدران شبه البيضوية الشكل المتناظرة المحور والمتشابهة الخواص في جميع الاتجاهات والممتلئة جزئياً أو كلياً بمائع ساكن ، عديم اللزوجة وغير قابل للانضغاط. وقد استخدمت طريقة (Rayliegh – Ritz) للحصول على الحل التقريبي والذي يطابق الحل الدقيق لحالات القشرية الفارغة والمملوءة بالكامل.

وقد تم استخدام (نظرية القشريات العميقة) لاختبار اهتزازات القشرية. وتعتمد هذه التقنية على أساس القشرية شبه البيضوية كمنظومة مستمرة مركبة من قشريتين نصف كرويتين متناظرتين على طول حدودها المستمرة.

تم عرض الحلول لبيان تأثير زاوية الإملاء للمائع على الترددات الطبيعية للقشرية. تم احتساب الترددات الطبيعية للقشرية في حالتيها الفارغة والمملوءة ووجد بان هده الترددات تقل بزيادة مستوى السائل داخل القشرية. وقورنت نتائج التحليل النظري مع النتائج المتيسرة للبحوث ذات العلاقة حيث أظهرت تطابقاً جيداً.

1. Introduction

Dynamic characteristics of oblate spheroidal shell filled with fluid are of great important in a variety of engineering applications ,such as, vibration of liquid oxygen tanks which are important components in upper stages space vehicles, and many other engineering and industrial systems. To show the resonance problem which is considered one of the important dynamic problems which results from these applications, free vibration of this type of shells are studied.

Although numerous papers have been written on the free vibration of oblate spheroidal shell, no work appears to have been done on the problem of fluid- filled isotropic oblate spheroidal shell. Nevertheless, there exists many papers in the case of spherical shell filled with fluid. Hoppmann[1]. Discussed both free and forced vibrations of a thin elastic orthotropic spherical shell, which is the general case of Love's spherical shell problem. Penzez and Burgin [2]. Discussed the problem of free vibration of thin isotropic oblate spheroid shell, Galerkin's method was employed. The effect of bending on vibration of spherical shell was reported by Kalinin's [3]. AL-Jumaily and Najim [4] considered the free vibration characteristics of an oblate spheroidal shell. They used Rayleigh variation method to obtain the natural frequencies and mode shape. Natural frequencies of an elastic hemispherical shell filled with a liquid and subjected to axisymmetric vibrations has been formulated by Tai and Wing [5]. The general non-axisymmetric free vibration of an isotropic elastic spherical shell filled with a compressible fluid medium is investigated by Chen and Ding [6]. Hayak & Dimaggio [7]. and Yen & Dimaggio [8] have considered the axisymmetric extensional motion of submerged spheroidal shell, free and forced, respectively. Engin and Lin [9] considered the free vibration of a thin, homogenous spherical shell containing fluid. The solution of vibration of a fluid-filled spherical membrane appears in Morse and Feshbach [10].Recently, free vibration of a thin spherical shell filled with a compressible fluid is investigated by Mingsian & Kuonung [11].

This study was undertaken to examine the effect of fluid filled on the vibration of thin oblate spheroidal shells. The comparison was made between the shell in cases of empty and filled with incompressible fluid like water. From the results, the changes of the natural frequencies of the mode of vibration will occur. Rayleigh – Ritz's method will be used to investigate the free vibration characteristics of this type of shells.

2. Equations of motion

The theoretical model in this paper consists of a thin elastic oblate spheroid shell under free, axisymmetric, non-torsional vibration filled with incompressible fluid. The analysis based on a non-shallow shell theory and Rayleigh-Ritz method to derive the equation of motion and then obtain the natural frequencies in cases of empty and filled shells.

2.1. Formulation of the problem

From the geometry shown in Figure (1) ,an oblate spheroidal shell is modeled as a structure composed of two spherical shells joined rigidly at their ends. Centers of curvature of their elements fall along the minor axis of the proposed structure. The radius of curvature at the apex of the shell (R_r) can be obtained from the geometrical formulation [2] :

$$R_\Phi = \frac{a(1-e^2)}{(1-e^2\cos^2\Phi')^{3/2}} \tag{1}$$

Setting (Φ') to zero results the radius of the shell at the apex as:

$$R_r = \frac{a}{(1-e^2)^{1/2}} \tag{2}$$

where,

$$e = \left[1 - \frac{b^2}{a^2}\right]^{\frac{1}{2}} \tag{3}$$

e = 0 for sphere , e = 1 for plate.

An approximate opening angle (Φ_0) may be obtained by using the following formula:

$$\Phi_o = \cos^{-1}\frac{R_r - b}{R_r} \tag{4}$$

Assuming that the temporal and spatail dependence of the free vibration are separable, the transverse displacement and the tangentail displacement may be assumed as [4]:

$$\begin{aligned} w(\Phi, t) &= W(\Phi) \,.\, \cos wt \\ u_\Phi(\Phi, t) &= U_\Phi(\Phi) \,.\, \cos wt \end{aligned} \tag{5}$$

Where (w) denotes the circular frequency, t: time and Φ denotes the angle measured from the (vertical axis). The actual Φ - dependent coefficient of the varaible was derived in Kalanins [3], as follows:

$$W(\Phi) = \sum_{i=1}^{3}(A_i P_{ni}(x) + B_i Q_{ni}(x)), \quad U_f(\Phi) = \sum_{i=1}^{3} -(1+n)D_i\left[A_i P'_{ni}(x) + B_i Q'_{ni}(x)\right] \tag{6}$$

Where

$$\left.\begin{aligned} &D_i = \frac{1+(l_i-2)/[(1+n)(1+x)]}{1-n-l_i+x(1-n^2)\Omega^2/(1+x)} \ , \ n_i = -0.5+\sqrt{0.25+l_i}\,, \\ &x = 12R_r^2/h^2, \quad x = \cos\Phi \end{aligned}\right\} \tag{7}$$

The parameters l_i's are the roots of the cubic equation :

$$l^3-\left[4+(1-n^2)\Omega^2\right]l^2+\left[4+(1-n)(1-n^2)\Omega^2+(1+x)(1-n^2)(1-\Omega^2)\right]l+ \\ (1-n)(1-n^2)\left[\Omega^2-2/(1-n)\right]\left[1+x(1+n)(\Omega^2+1/(1+n))\right]=0 \tag{8}$$

The non-dimensional frequency is defined as:

$$\Omega^2 = rw^2R_r^2/E \tag{9}$$

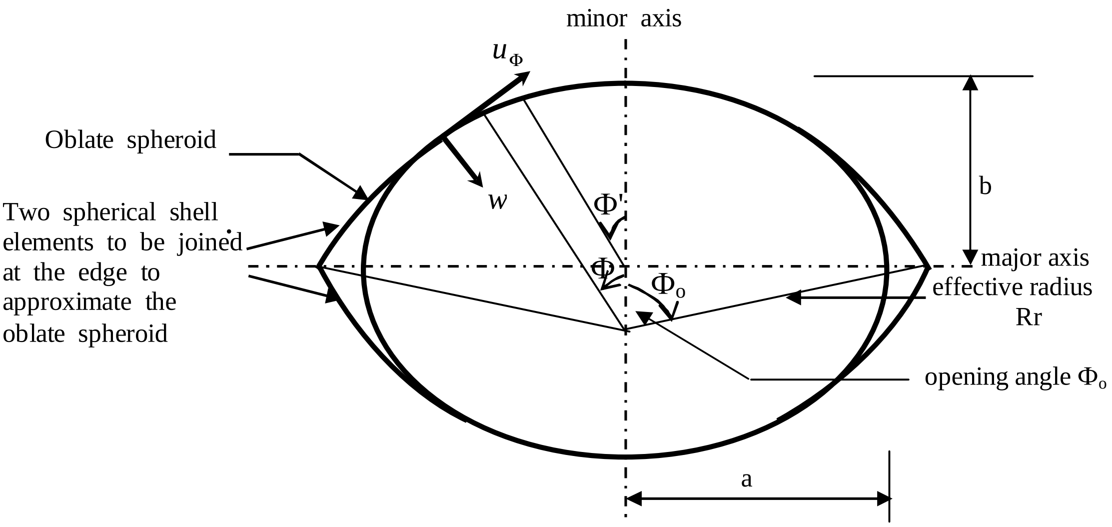


Figure (1). An oblate spheroid and its approximate of two spherical shell elements joined at the edge.

2.2 Energy method

Because of the complexity encountered in solving the exact equation of motion of an oblate spheroidal shell, an approximate energy approach based on Rayleigh-Ritz's method is used. Rayleigh-Ritz's method is an extension of Rayleigh's quotient which can be used for more complex elastic bodies and helps to determine the natural frequencies and their associated mode shapes with general boundary conditions in an approximate form. In order to apply Rayleigh method, and its extension, the Rayleigh-Ritz's procedure, we need to derive expression for the maximum kinetic and potential energies. Physically, the frequency oscillation is found from the ratio of these energies [12].

The kinetic energy of the system is defined to be:

$$T = T_s + T_f \tag{10}$$

Where the kinetic energy T_S of an oblate spheroidal shell is:

$$T_S = \int_{-h/2}^{h/2} \int_0^{2p} \int_0^{2p} \tfrac{1}{2}\, r \left[\left[u^{\bullet}{}_f \right]^2 + \left[w^{\bullet} \right]^2 \right] R_\Phi\, R_q \sin\Phi' d\Phi' dq\ dz \tag{11}$$

And the kinetic energy T_f of incompressible ,non-viscous, irrotational fluid is given by:

$$T_f = 2\int_0^{r} \int_0^{2p} \int_0^{a} r_{\ f} \left[w^{\bullet} \right]^2 R_\Phi\, R_q \sin\Phi' d\Phi' da\ dr \tag{12}$$

The dot indicates a time derivative.

The strain energy of the shell is given by [13]:

$$U = \int_{-h/2}^{h/2} \int_0^{2p} \int_0^{2p} \frac{1}{2} [б_{\Phi'}\ Є'_{\Phi'} + б_{\theta} Є'_{\theta}]\, R_\Phi\ R_\theta \sin\Phi'\, d\Phi\, d\theta\, dz \tag{13}$$

The stress in terms of strain are defined as [13]:

$$\left.\begin{aligned} &б_{\Phi'} = \frac{E}{(1-v^2)}[Є'_{\Phi'} + \upsilon Є'_{\theta}]\ ,\quad б_{\theta} = \frac{E}{(1-v^2)}[Є'_{\theta} + \upsilon Є'_{\Phi'}] \\ &\text{and} \\ &Є'_{\Phi'} = Є^{\circ}{}_{\Phi} + Z K_\Phi \qquad ,\quad Є'_{\theta} = Є^{\circ}{}_{\theta} + Z K_\theta \end{aligned}\right\} \tag{14}$$

$$\left.\begin{aligned} &\epsilon^{\circ}_{\theta} = \frac{1}{R_q}\left[u_f \cot f' + w\right] \quad , \quad \epsilon^{\circ}_{\Phi} = \frac{1}{R_f}\left[\frac{\partial u_f}{\partial f'} + w\right] \\ &K_{\theta} = \frac{\cot f'}{R_f^{\,2}}\left[(u_f - \frac{\partial w}{\partial f'})\right] \quad , \quad K_{\Phi} = \frac{1}{R_f}.\frac{\partial}{\partial f'}\left[\frac{1}{R_f}(u_f - \frac{\partial w}{\partial f'})\right] \end{aligned}\right\} \tag{15}$$

At the natural frequency (ω), and assuming separation of variables, the shell displacements may be written in the following forms [13].

$$\left.\begin{aligned} &w(\Phi', t) = W(\Phi') \, . \, e^{i\omega t} \\ &\text{and} \\ &u_{\Phi}(\Phi', t) = U_{\Phi}(\Phi') \, . \, e^{i\omega t} \end{aligned}\right\} \tag{16}$$

Taking e^{iwt} in Eqs. (16) to be unity and integrating the equations (11) and (12) with respect to (z) and (r), respectively, the maximum kinetic energy of the system will take the form:

$$\left.\begin{aligned} T_{max} = \frac{w^2 r\ h}{2}\int_0^{2p}\int_0^{2p} (U_{\Phi}^2 + W^2) R_{\Phi} R_{\theta} \sin\Phi' \, d\Phi' \, d\theta \; + \\ 2rw^2 r_f \int_0^{2p}\int_0^{a} (W^2) R_{\Phi} R_{\theta} \sin\Phi' \, d\Phi' \, d a \end{aligned}\right\} \tag{17}$$

Where $r = R_r$ and $R_q = \dfrac{a}{(1-e^2\cos^2\Phi')^{1/2}}$

Substituting equations (14) , (15) and (16) in equation (13), the maximum strain energy of the shell is obtained after performing the integration with respect to (z) and taken e^{iwt} to be unity:

$$
\begin{aligned}
U_{\max} = \frac{Eh}{2(1-u^{2})} \int_{0}^{2p} \int_{0}^{2p} \Bigg\{ & \frac{h^{2}}{12} \left[\frac{1}{R_{\Phi}^{2}} \left[\frac{\partial}{\partial \Phi'} \left[\frac{U_{\Phi}}{R_{\Phi}} - \frac{\partial W}{R_{\Phi}\, \partial \Phi'} \right] \right]^{2} \right. \\
& + \frac{\cos^{2}\Phi'}{R_{\Phi}^{2}\, R_{q}^{2}\, \sin^{2}\Phi'} \left[U_{\Phi} - \frac{\partial W}{\partial \Phi'} \right]^{2} + 2u\, \frac{\cos\Phi'}{R_{q}\, R_{\Phi}^{2}\, \sin\Phi'} \left[U_{\Phi} - \frac{\partial W}{\partial \Phi'} \right]. \\
& \left. \cdot \frac{\partial}{\partial \Phi'} \left[\frac{U_{\Phi}}{R_{\Phi}} - \frac{\partial W}{R_{\Phi}\, \partial \Phi'} \right] \right] + \frac{1}{R_{\Phi}^{2}} \left[\frac{\partial U_{\Phi}}{\partial \Phi'} + W \right]^{2} \\
& + \frac{1}{(R_{q}\, \sin\Phi')^{2}} \left(U_{\Phi} \cos\Phi' + W \sin\Phi' \right)^{2} \\
& + \frac{2u}{R_{q}\, R_{\Phi} \sin\Phi'} \left[\frac{\partial U_{\Phi}}{\partial \Phi'} + W \right] . \left(U_{\Phi} \cos\Phi' + W \sin\Phi' \right) \Bigg\}. \\
& R_{\Phi}\, R_{q}\, \sin\Phi'\, d\Phi'\, dq
\end{aligned}
\tag{18}
$$

3. Frequency equation

For a system with no dissipation losses, as those due to friction or damping, the maximum potential energy equals the maximum kinetic energy, i. e.

$$U_{\max} = T_{\max} \tag{19}$$

The kinetic energy for ω=1 rad/sec is customarily define as T^{*}_{max} and, therefore,

$$T_{\max} = \omega\, T^{*}_{\max} \tag{20}$$

An expression for the natural frequency may be written as :

$$w_i^{2} = \frac{U_{\max}}{T^{*}_{\max}} \qquad i = 1, 2, 3, \ldots, n \tag{21}$$

Following the procedure of Rayleigh – Ritz's method, the radial (or transverse) and tangential displacements can be written in power series form as :

$$w(\Phi') = \sum_{i=1}^{n} a_i . W_i(\Phi') , \quad u_{\Phi}(\Phi') = \sum_{i=1}^{n} b_i . U_{\Phi i}(\Phi') \tag{22}$$

where a_i and b_i are coefficients to be determined. The functions $w(\Phi')$, $u_\Phi(\Phi')$ satisfy all the geometry boundary conditions of the system. Equation (21) is an exact expression for the frequency according to Rayleigh quotient. In order to use the procedure of Rayleigh – Ritz's method, equation (22) is substituted into equation (17) and (18), then the results are used in equation (21). After some mathematical manipulations, the following equation will result:

$$w_i^{\,2} = \frac{U_{\max}}{T^*_{\max}} = \frac{\sum_{i=1}^{n}\sum_{j=1}^{n} c_i\, c_j\; k_{ij}}{\sum_{i=1}^{n}\sum_{j=1}^{n} c_i\, c_j m_{ij}} \qquad i = 1, 2, 3, \ldots.., n \tag{23}$$

where,

$$\begin{aligned} U_{\max} = \sum_{i=1}^{n}\sum_{j=1}^{n} c_i\, c_j \frac{Ehp}{(1-u^2)} \int_0^{2p} \Big\{ & \frac{h^2}{12R_\Phi^4}\left[U'_{\Phi i}\, U_{\Phi j}' - 2U_{\Phi i}'\, W_i'' + W_i''\, W_j''\right]\sin\Phi' \\ & + \frac{u\, h^2}{6R_q\, R_\Phi^3}\left[U_{\Phi i}\, U_{\Phi i}' - U_{\Phi i}\, W_i'' - U_{\Phi i}'\, W_i' + W_i'\, W_i''\right]\cos\Phi' \\ & + \frac{h^2}{12R_\Phi^2\, R_q^2}\left[U_{\Phi i} U_{\Phi j} - 2U_{\Phi i}\, W_i' + W_i'\, W_j'\right]\frac{\cos^2\Phi'}{\sin\Phi'} \\ & + \frac{1}{R_\Phi^2}\left[U_{\Phi i}'\, U_{\Phi j}' + 2U_{\Phi i}'\, W_i + W_i\, W_j\right]\sin\Phi' \\ & + \frac{1}{R_q^2}\left[U_{\Phi i}\, U_{\Phi j}\, \frac{\cos^2\Phi'}{\sin\Phi'} + 2U_{\Phi i}\; W_i \cos\Phi' \; + W_i\, W_j \sin\Phi' \right] \\ & + \frac{2u}{R_\Phi\, R_q}\left[U_{\Phi i}\, U_{\Phi i}'\; \cos\Phi' \; + U_{\Phi i}'\, W_i \sin\Phi' \; + U_{\Phi i}\, W_i \cos\Phi' \; + W_i\, W_i \sin\Phi' \right] \Big\}. \\ & . R_\Phi\, R_q\, d\Phi' \end{aligned} \tag{24}$$

and

$$\begin{aligned} T^*_{\max} = & \sum_{i=1}^{n}\sum_{j=1}^{n} c_i\, c_j \int_0^{2p} r\;\, hp\, \left[U_i\, U_j + W_i\, W_j\right] R_\Phi\, R_q\; \sin\Phi'\, d\Phi' + \\ & \sum_{i=1}^{n}\sum_{j=1}^{n} c_i\, c_j \int_0^{2p} 2rr_f\, a\,(W_i\, W_j\,)\; R_\Phi\, R_q \sin\Phi'\, d\Phi' \qquad . \end{aligned} \tag{25}$$

Equations(24) and (25) gives the physical properties of the shell from the stiffness and mass distribution point of view. The stiffness and mass of the shell are given by the following two equations respectively:

$$\begin{aligned}
k_{ij} = \frac{Ehp}{(1-u^2)} \int_0^{2p} \Big\{ & \frac{h^2}{12R_\Phi^4}\left[U_{\Phi i}'\ U_{\Phi j}' - 2U_{\Phi i}' W_i'' + W_i'' W_j''\right]\sin\Phi' \\
& + \frac{u\, h^2}{6R_q R_\Phi^3}\left[U_{\Phi i} U_{\Phi i}' - U_{\Phi i} W_i'' - U_{\Phi i}' W_i' + W_i' W_i''\right]\cos\Phi' \\
& + \frac{h^2}{12R_\Phi^2 R_q^2}\left[U_{\Phi i} U_{\Phi j} - 2U_{\Phi i} W_i' + W_i' W_j'\right]\frac{\cos^2\Phi'}{\sin\Phi'} \\
& + \frac{1}{R_\Phi^2}\left[U_{\Phi i}' U_{\Phi j}' + 2U_{\Phi i}' W_j + W_i\ W_j\right]\sin\Phi' \\
& + \frac{1}{R_q^2}\left[U_{\Phi i} U_{\Phi j}\frac{\cos^2\Phi'}{\sin\Phi'} + 2U_{\Phi i}\ W_i \cos\Phi' + W_i W_j \sin\Phi'\right] \\
& + \frac{2u}{R_\Phi R_q}\left[U_{\Phi i} U_{\Phi i}'\cos\Phi' + U_{\Phi i}' W_i \sin\Phi' + U_{\Phi i} W_i \cos\Phi' + W_i W_i \sin\Phi'\ \right] \Big\}. \\
& . R_\Phi R_q\, d\Phi'
\end{aligned} \tag{26}$$

and

$$\begin{aligned}
m_{ij} = & \int_0^{2p} r\ hp\ \left[U_i U_j + W_i W_j\right] R_\Phi R_q\ \sin\Phi'\ d\Phi' + \\
& \int_0^{2p} 2rr_f\ a\ (W_i W_j\ )\ R_\Phi R_q \sin\Phi'\ d\Phi' \qquad .
\end{aligned} \tag{27}$$

In order to minimize the approximate value, which is given by equation (23), it should be differentiated with respect to c_i and equating the resulting expression to zero, that is :

$$\frac{\partial w^2}{\partial c_i} = \frac{T_{\max}^* \dfrac{\partial U_{\max}}{\partial c_i} - U_{\max}\dfrac{\partial T_{\max}^*}{\partial c_i}}{T^{*2}_{\max}} = 0 \qquad \text{i=1,2,3 } \ldots\ldots\ldots \text{n} \tag{28}$$

This equation can be satisfied if and only if the numerator equal zero, since $T^*_{\max}$ is never equal to zero. The numerator can be written in a more useful form as:

$$\frac{\partial U_{\max}}{\partial c_i} - \frac{U_{\max}}{T^*_{\max}} \frac{\partial T^*_{\max}}{\partial c_i} = 0 \qquad i = 1, 2, 3, \ldots, n \tag{29}$$

It is as given by equation (21), $w_i = \frac{U_{\max}}{T^* \max}$, and n is the number of terms in the approximate solution. The infinite degrees of freedom system has been replaced by an n degrees of freedom system. Therefore, Equation (28) can be written in general matrix notation as :

$$\left[\{K\} - w^2\{M\}\right]\{c\} = \{0\} \tag{30}$$

The stiffness and mass are matrices determined at the edge ($\Phi = \Phi_0$) of the sphereical shell using (Eqs. 26, 27) respectively, which resulted values substituted in the following determinant:

$$\begin{vmatrix} k_{11} - \Omega^2 m_{11} & k_{12} - \Omega^2 m_{12} & k_{13} - \Omega^2 m_{13} \\ k_{21} - \Omega^2 m_{21} & k_{22} - \Omega^2 m_{22} & k_{23} - \Omega^2 m_{23} \\ k_{31} - \Omega^2 m_{31} & k_{31} - \Omega^2 m_{32} & k_{33} - \Omega^2 m_{33} \end{vmatrix} = 0 \tag{31}$$

4. Calculation of natural frequency

The calculation of the natural frequency is carried out by specifying an intial guess then evaluting the determiniant of equation (31). Increasing the frequency by small increments and repeating the same procedure until the value of the determinant changes in sign. This indicates that a natural frequency has a new value. The frequency increment is then minimized and the operation is repeated until the desired accuracy of the non-dimentional natural frequency is obtained when the determinant is vanished.

In case of zero filling angle (a) in equations (17) ,natural frequencies for an empty shell can be calculated using the same procedure of Rayleigh –Ritz method.

5. Results and discussion

Calculations to test the theory in the case of partially or completely filled shells are presented herein. The parameters used in this study are: a=0.185 mm, h=0.0015mm, $n = 0.3$, $E = 68GN/m^2$, e = 0.683, $r_s = 2720kg/m^3$, and $r_f = 1000kg/m^3$. The parameters used here for the empty shell are the same as in refrence[4].

For validity purposes, a comparison is made between the theoretical results obtained by Rayleigy-Ritz method (RRM) in this paper and the boundary maching method (BMM) in AL-Jumaily and Najim [4] with some experimental results which are taken from AL-Jumaily and Najim [4] for the case of filling angle ($a = 0$). There is an excellent coincidence between the theoretical results of RRM and BMM method. However, the theoretical results obtained by RRM are higher than experimental results, because the theoretical spherical caps are in general stiffer than the corresponding experimental oblate spherical shells.. See Table (1).

Table (1) . Theoretical and experimental natural frequencies in Hz of thin oblate spheroidal shell (α = 0) .

Frequency	BMM[4]	RRM (Preset work)	Experimental [4]
1	2500	2520	2400
2	2978	2973	2600
3	3082	3090	2900
4	3180	3190	3100

Figures (1) and (2) show the non-dimensional natural frequencies ($l = \sqrt{E/r}.w.a$) of the first two modes of vibration as functions of the eccentricity ratio of empty and filled shells by the Rayleigh- Ritz method using the non-shallow shell theory. These figures show that when the eccentricity increases, the natural frequency decreases, whereas that of an empty shell is higher than those of filled shell with incompressible fluid. This drop-off in natural frequencies can be clearly justified according to the increase in the fluid induced mass.

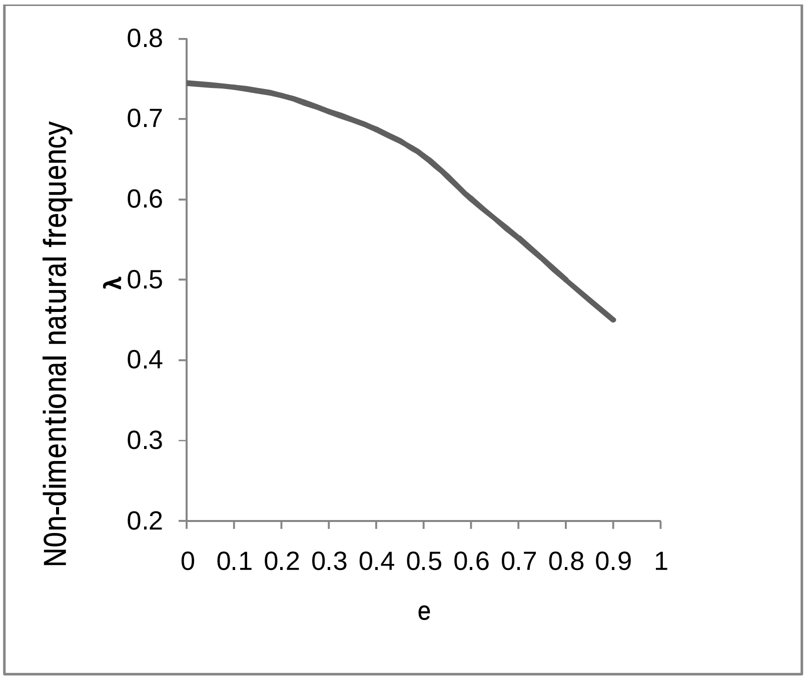


Figure (1) . Effect of eccentricity onthe first bending modes for zero filling angle .

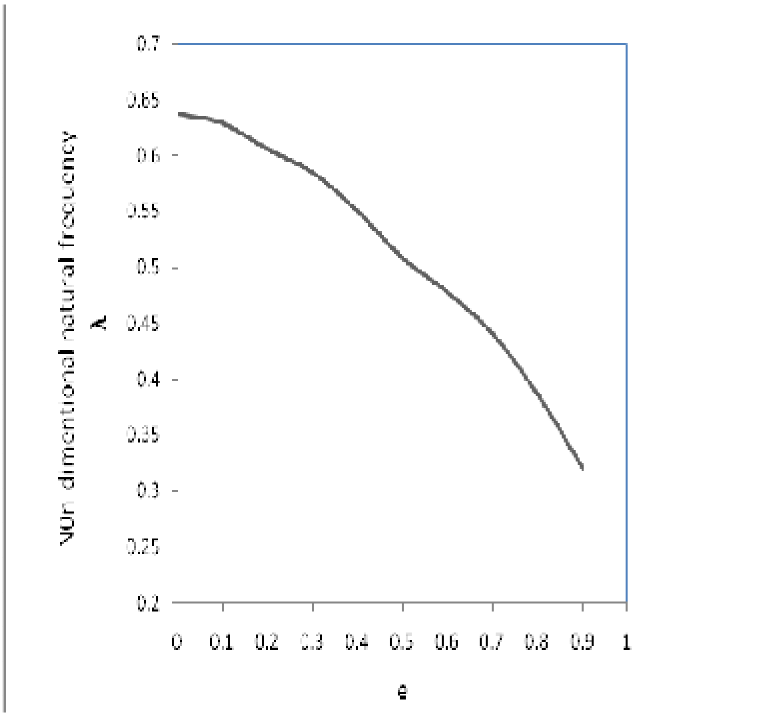


Figure (2) . Effect of eccentricity onthe first bending modes for fluid filling shells .

The free vibrations of the shell is absolutely affected by the fluid mass and thus the wet angle parameter α plays a main role in this study. The angle of filling , α , takes the values $\alpha = 0,60,120,180,240,300,360^{o}$ and the corresponding natural frequencies were computed. Figure (3) shows the decrease in the natural frequency when the fluid level in the shell increased.

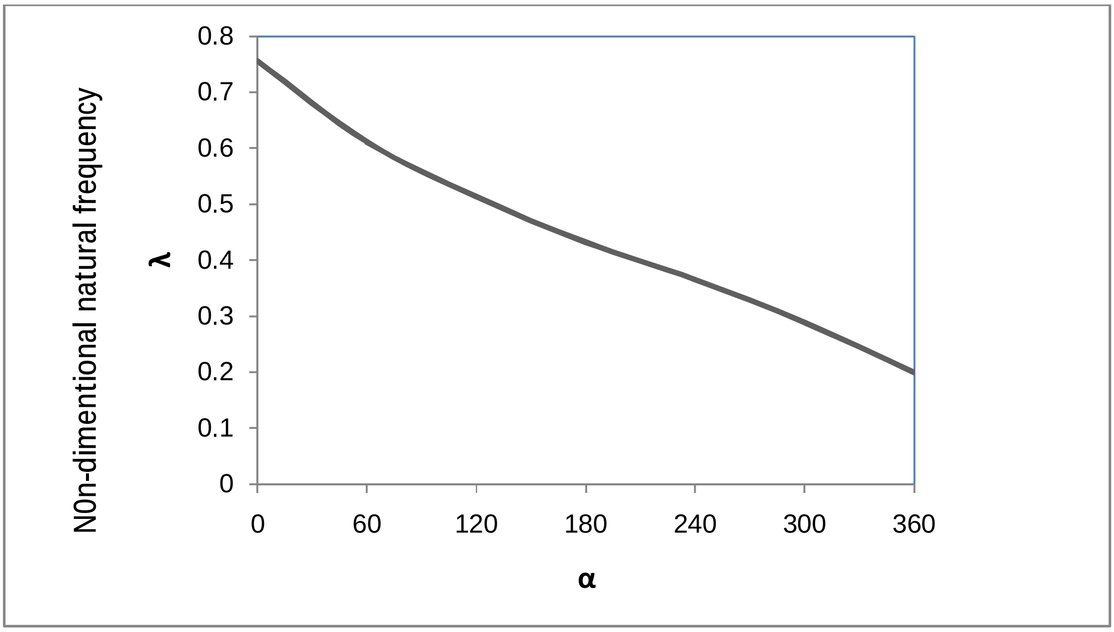
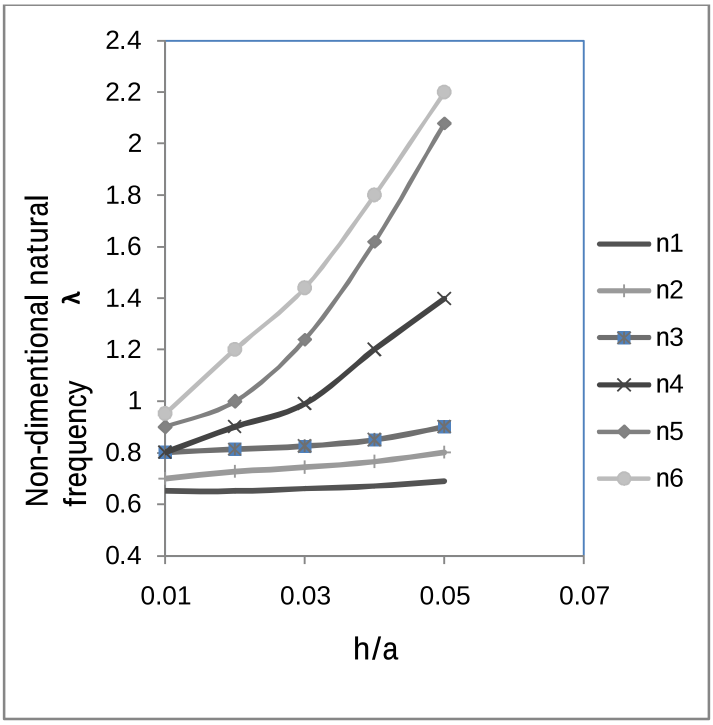
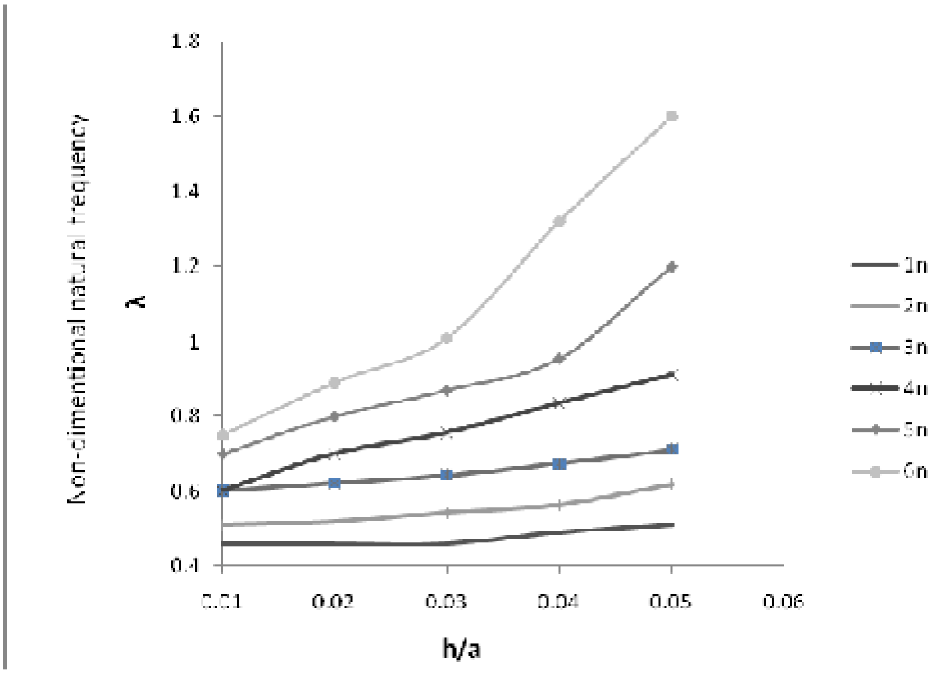


Figure (3). Variation of the non-dimentional natural frequency of the oblate spheroid shell varies angle of filling.

The effect of the shell thickness on the free vibration characteristics of the empty and filled shell is investigated in Figs. (4) and (5). It can be noted that the variation of the natural frequency of the bending modes increases with increases of thickness ratio. This phenomena can be elaborated due to the fact that the strain energy increased with increasing the thickness ratio.

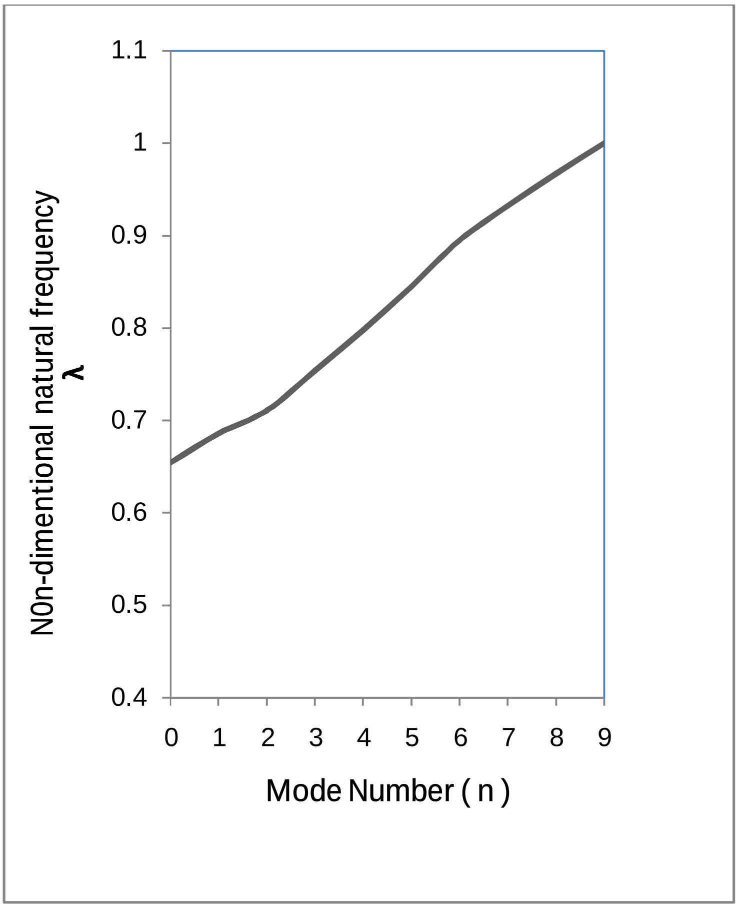


Figure(4) . Effect of the thickness ratio on the natural frequency of the oblate spheroid shell (e= 0.6, α = 0) .

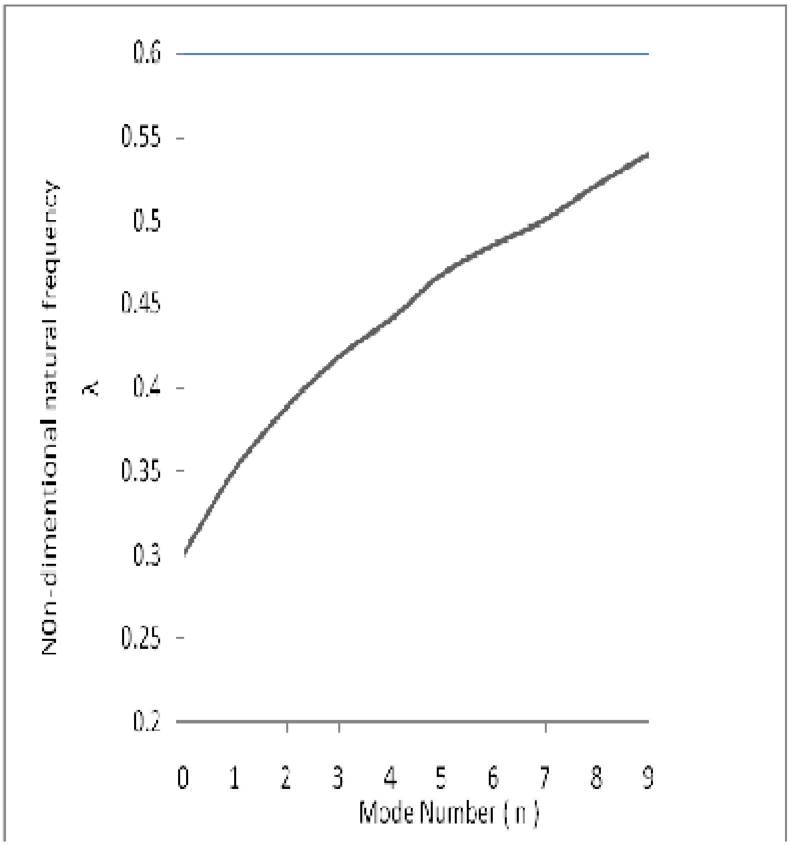


Figure(5) . Effect of the thickness ratio on the natural frequency of the oblate spheroid shell (fluid filled ,e= 0.6) .

For a different mode number (n), it is seen that as (n) increases, the natural frequency increases too. It is found that the presence of fluid decreases the natural frequency, and this can be explained as the system-mass increases, as shown in Figures (6) and(7).



Figure(6) . Variation of the non-dimentional natural frequency of the oblate spheroid shell varies mode number of empty shell (e = 0.6).



Figure(7) . Variation of the non-dimentional natural frequency of the oblate spheroid shell varies mode number of filled shell (e = 0.6).

6. Conclutions

Free axisymmetric vibrations of thin isotropic oblate spheroidal shell containing incompressible fluid has been studied, to show the effect of fluid on its dynamic characteristics using non-shallow shell theory and Rayleigh-Ritz method .Numerical analysis resulted the following conclusion:

1- The kinetic energy of the system increases with the consequence that natural frequencies decrease, as can be seen from the Rayleigh-Ritz method.

2- The natural frequencies of the fluid-filled oblate shell are lower than those of the empty shell parameters of the oblate spherical itself, i.e. the fluid has effects on the frequencies. This result affects engineering design.

3- The analytical approach used in this paper is relativly simple and doesn't need advanced computer system, thus, it can be implementry in many engineering applications such as tank's design.

4- The resultes show the resonance frequencies of the fluid filled shell decreased with the increase of fluid level in the shell.

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8. Nomenclature

a	Major semi – axis of an oblate spheroid shell.
b	Minor semi – axis of an oblate spheroid shell.
E	Young's modulus of elasticity (GN / m^2).
e	Eccentricity ratio.
h	Shell thickness (mm).
r	Radius of spherical shell (mm)
$P_n(x)$	Legendre function of the first kind.
$P'_n(x)$	First derivative of the Legendre function of the first kind.
$P''_n(x)$	Second derivative of the Legendre function of the first kind.
R_r	Effective radius (mm)
R_Φ , R_θ	Principal radii of curvatures of an oblate spheroid.
U_Φ	Tangential displacement mode.
u_Φ	Tangential displacement of points on shell middle surface.
W	Transverse displacement mode.
w	Transverse displacement of points on shell middle surface.
$\epsilon_\Phi,\epsilon_\theta,\epsilon_r$	Strains
Φ'	Inclination angle of an oblate spheroid.
Φ	Inclination angle of a spherical shell model.
Φ_o	Opening angle of the approximate spherical shell.
λ	Non – dimensional frequency parameter (($\rho / E)^{1/2} \omega.a$). (used for oblate spheroid shells)
θ	Angle of rotation in the meridian direction.
r_s	Density of shell(kg / m^3).
r_f	Density of fluid(kg / m^3).
a	Angle of filling .
Ω	Non – dimensional frequency parameter (($\rho / E)^{1/2} \omega.R$). (used for spherical shells)
ω	Circular frequency (rad / sec)