Study of Mixed Convection Heat Transfer Inside a Vented Square Cavity with Inner Heated Cylinder

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Abstract

The problem of mixed convection heat transfer for laminar air flow in a square cavity with inner heated cylinder is numerically analyzed by finite element method using software package (FlexPDF) to solve the conservation of governing equations. The right vertical wall is kept at constant temperature and the others are adiabatic. An external flow enters the cavity through an opening in the left vertical wall and exits from another opening in the right vertical wall. Results of streamlines, isotherms, average temperature and average Nusselt number of the heated wall are presented for Ri = 0 to 12, $50 \leq Re \leq 200$, Pr = 0.71 and inlet position hi = 0.2. From the present analysis it is found that with increases of Re and Ri numbers the convective heat transfer becomes predominant over the conduction heat transfer. The results of streamlines and isotherms are compared with available result of Rahman et al.[8], a good agreement has been achieved.

Keywords : Heat transfer , mixed convection , fluid flow .

دراسة عملية انتقال الحرارة بالحمل المختلط داخل فجوة مربعة الشكل تحوي اسطوانة داخلية مسخنة

المستخلص

مسألة انتقال الحرارة بالحمل المختلط لجريان الهواء الطباقي داخل فجوة مربعة تحوي اسطوانة داخلية مسخنة تم تحليلها عدديا بطريقة العناصر المحددة باستخدام الحقيبة البرمجية (FlexPDF) لحل معادلات الحفظ الحاكمة. الجدار العمودي الأيمن للفجوة مثبت عند درجة حرارة ثابتة إما الجدران الأخرى فتكون معزولة . الجريان الخارجي يدخل الفجوة من خلال فتحة موجودة في الجدار العمودي الأيسر ويخرج عن طريق فتحة مثبتة بالجدار العمودي الأيمن. النتائج تمثلت بخطوط الجريان، خطوط ثابتة درجة الحرارة ، معدل درجة الحرارة ومعدل رقم نسلت عند رقم ريتشارد Ri يتراوح من (0-12) ورقم رينولدز $50 \leq Re \leq 200$ ، رقم برانتل Pr = 0.71 عند موقع فتحة الدخول hi = 0.2. أظهرت النتائج

الحالية بأنه مع زيادة Re و Ri فان عملية انتقال الحرارة بالحمل سائدة على عملية انتقال الحرارة بالتوصيل. نتائج خطوط الجريان وخطوط التحارر قورنت مع ما منشور في Rahman et al. [8] وأظهرت توافقا جيدا.

1. Introduction

Mixed convection flows are present in many transport processes in nature and in engineering devices. Applications of mixed convection flows can be found in heat exchangers, nuclear reactors, solar energy storage, heat rejection systems, heaters and refrigeration devices, etc .Various geometries of fluid-filled rectangular enclosures have been theoretically and experimentally modeled in order to look at the effects of some design parameters on the thermal performance of simulated systems.

Thermal buoyancy forces play a significant role in forced convection heat transfer when the flow velocity is relatively small and the temperature difference between the surface and the free stream is relatively large. The buoyancy force modifies the flow and the temperature fields and hence the heat transfer rate from the surface. Problems of heat transfer in enclosures by free convection or combined free and forced convection have been the subject of investigations for many years. The influence of the physical characteristics of heat sources on the mixed convection heat transfer performance has been examined by many researchers. Kumar and Yuan [1] studied the laminar, two-dimensional mixed convection flow in a rectangular enclosure with inlet and outlet ports. Papanicolaou and Jaluria [2] investigated mixed convection in a cavity with a localized heat source.

Ajay and Jeffrey [3] studied mixed convection within a recirculating flow in an insulated lid-driven cavity of rectangular cross section (150mm X 450mm) and depth varying between 150mm and 600 mm ,by appropriately varying the lid speed ,the vertical temperature differential ,and the depth by taking Gr/Re^2 ratios from 0.1 to 1000 .The mean flux values over the entire lower boundary were analyzed to produce Nusselt number and Stanton number correlations which should be useful for design applications. Raji and Hasnaoui [4] reported the results of a numerical study of air laminar mixed convection in a rectangular cavity, including radiation, for $10^3 \leq Ra \leq 5X10^6$ and $5 \leq Re \leq 5000$.

Mixed convection in an open cavity with a heated wall bounded by a horizontally insulated plate was studied by Oronzio et al. [5], three basic heating modes are considered: the heated wall is on the inflow side, the heated wall is on the opposing flow, and the heated wall is the horizontal surface of the cavity (heating from below). Manca et al. [6] worked out a numerical investigation of three different cases of mixed convection in a channel with an open cavity for various ratios of channel opening and cavity height and the range of governing parameters were $0.1 \leq Ri \leq 100$, and Re=100 and 1000. Investigated a mixed convection problem in a two sided lid–driven and differentially heated cubic for Ri number range $0.01 \leq Ri \leq 10$.

The finite-element procedure based on the projection method for solving the primitive variables form of the Navier-Stokes equations and energy equation in three dimensions, proposed by Lo et al [7], used numerical algorithm involves the operator splitting technique, balance tensor diffusivity (BTD), the Runge-Kutta time-stepping method, and a bi-conjugate gradient iterative solver. Two benchmark flows, lid-driven cavity flow and laminar flow over a 3-D backward-facing step, are validated with numerical solutions. Mixed-convection problem in a two-sided lid-driven and differentially heated cubic cavity is investigated. The Richardson number in the range $0.01 \leq Ri \leq 10$, serves as a measure of the relative importance of forced- and natural-convection modes on the heat transfer.

Rahman et al. [8] investigated mixed convection in a vented enclosure. An external fluid flow enters the enclosure through an opening in the left vertical wall and exits from another fixed opening in the right vertical wall. For mixed convection, the significant parameters are Grashof number (Gr), Richardson number (Ri) and Reynolds number (Re) by which different fluid and heat transfer characteristics inside the cavity are obtained .Results show that with the increase of Re and Ri the convective heat transfer become dominant. Behzad et al. [9] studied the numerically investigate the cooling performance of electronic devices with an emphasis on the effects of the arrangement and number of electronic components. The analysis uses a two dimensional rectangular enclosure under combined natural and forced convection flow conditions and considers a range of Rayleigh numbers. Rahman et al. [10] studied the mixed convection in cavity contains a heat conduction horizontal square block located inside the cavity. The investigations are conducted for various values of geometric size, location and thermal conductivity of the block under constant Re and Pr. The results indicated that the average Nusselt number and the temperature at the center of solid block are strong dependet on the system configurations studied under different geometrical and physical conditions. The objective of this study to analyze the effects of Richardson and Reynolds numbers and the location of the inner heated cylinder on the heat transfer characteristics of laminar mixed convection. Finite element method has been used to solve the governing equations by using software package (FlexPDF).

2. Model Description

Details of the geometry for the configuration are shown in Figure (1). The model considered here is a square cavity consists of inner isothermal cylinder. The uniform constant temperature T_h, applied on the right vertical wall while the other walls are assumed to be adiabatic. The inflow opening located on the left vertical wall at a distance hi=0.2 from the bottom of the enclosure. The outflow opening of the cavity is fixed at the top of the opposite heated wall and the size of the inlet port is the same for the exit port which is equal to W = 0.1L. It is assumed that the incoming flow is at a uniform velocity, u_i and at the ambient temperature, T_i. Since the boundary conditions at the exit of the cavities are unknown, values of u, v and T are extrapolated at each iteration step. All solid boundaries are to be rigid no-slip walls.

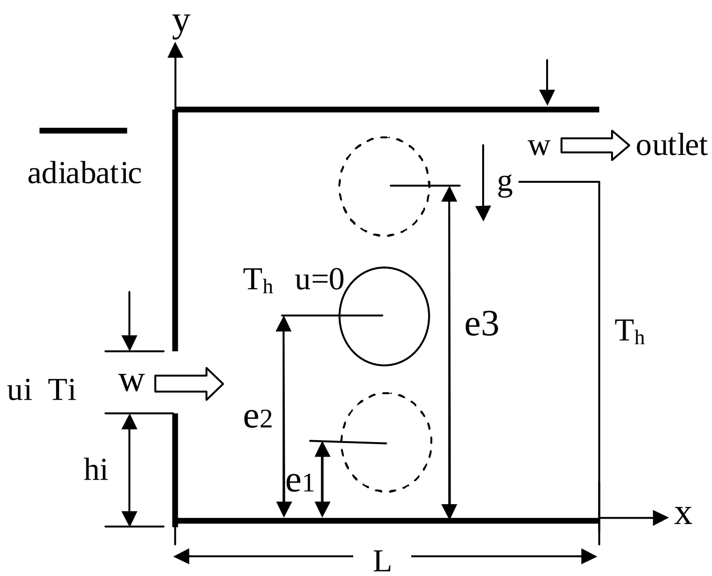


Figure (1). Schematic diagram of the problem considered and boundary conditions.

3. Governing equations

Mixed convection is governed by the differential equations expressing conservation of mass, momentum and energy. The present flow is considered steady, laminar, incompressible and two-dimensional. The viscous dissipation term in the energy equation is neglected. The physical properties of the fluid in the flow model are assumed to be constant except the density variations causing a body force term in the momentum equation. The governing equations for steady mixed convection flow can be expressed in the dimensionless form as:

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0 \tag{1}$$

$$U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}=-\frac{\partial P}{\partial X}+\frac{1}{\mathrm{Re}}(\frac{\partial^2 U}{\partial X^2}+\frac{\partial^2 U}{\partial Y^2}) \tag{2}$$

$$U\frac{\partial V}{\partial X}+V\frac{\partial V}{\partial Y}=-\frac{\partial P}{\partial Y}+\frac{1}{\mathrm{Re}}(\frac{\partial^2 V}{\partial X^2}+\frac{\partial^2 V}{\partial Y^2})+Ri\,\theta \tag{3}$$

$$U\frac{\partial \theta}{\partial X}+V\frac{\partial \theta}{\partial Y}=\frac{1}{\mathrm{Pr\,Re}}\left(\frac{\partial^2 \theta}{\partial X^2}+\frac{\partial^2 \theta}{\partial Y^2}\right) \tag{4}$$

Where X and Y are the coordinates varying along horizontal and vertical directions respectively, U and V are the velocity components in the X and Y directions respectively, θ is the dimensionless temperature and P is the dimensionless pressure.

The dimensionless parameters in the above equations can be given as:

$$X=\frac{x}{L},\quad Y=\frac{y}{L},\quad U=\frac{u}{u_i},\quad V=\frac{v}{u_i},\ \theta=\frac{T-T_i}{T_h-T_i},\quad P=\frac{p}{\rho u_i^{\,2}}$$

$$\mathrm{Pr}=\frac{\upsilon}{\alpha}\quad,\quad Ri=\frac{g\beta(T-T_i)L}{u_i^{\,2}}\quad,\quad \mathrm{Re}=\frac{L\,u_i}{\upsilon}$$

Where ρ, β, ν, α and g are the fluid density, coefficient of volumetric expansion, kinematics viscosity, thermal diffusivity, and gravitational acceleration, respectively. The following boundary conditions are used:

Inlet: U = 1, V = 0, θ = 0

Exit: Convective boundary condition, P = 0

At the cavity walls (except the right vertical wall): U=V=0, $\partial\theta/\partial n=0$

At the heated right vertical wall: U=V=0, θ=1

At the cylinder U=V=0, θ=1

The local Nusselt number at the heat wall is defined as:

$$Nu_L=\frac{\partial\theta}{\partial n} \tag{5}$$

The average Nusselt number

$$\overline{Nu} = \frac{1}{L_h}\int_0^{L_h} Nu_L ds \qquad (6)$$

And the bulk average temperature in cavity is defined as

$$\theta_{av} = \int \frac{1}{\overline{V}}\theta d\overline{V} \qquad (7)$$

Where L_h is the length of the heat wall and $\overline{V}$ is the cavity volume.

4. Numerical solution

The code FlexPDE is used to perform the numerical calculations using finite element method to analyze the laminar mixed convection heat transfer and fluid flow in a vented square cavity contain of inner heated cylinder. It is well known in the numerical solution field that the set of equations above (1-4) may be highly oscillatory or even sometimes undetermined because of inclusion of the pressure term in the momentum equations. In finite element method there is a derived approach with purpose of stabilizing pressure oscillations and allowing standard grids and elements. This approach enforces the continuity equation and the pressure to give the following, what called, penalty approach [11].

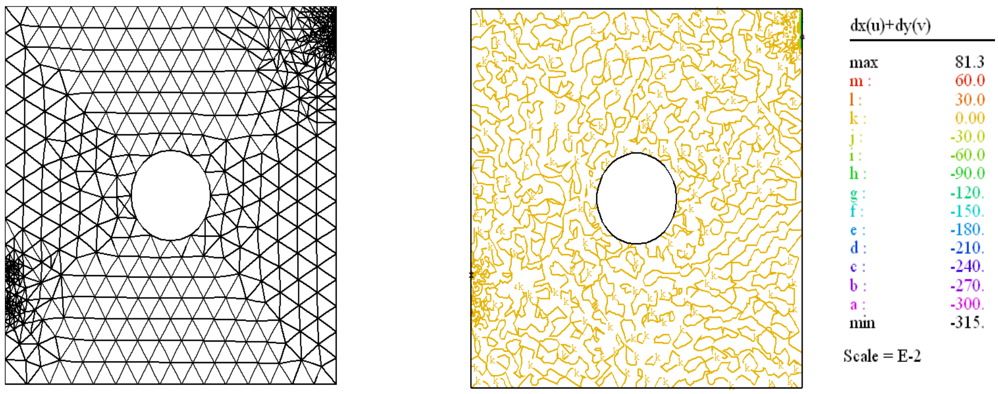
$$\nabla^2 P = \gamma\left(\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y}\right) \qquad (8)$$

Where γ is a parameter that should be chosen either from physical knowledge or by other means [11]. A most convenient value for γ was attained in this study to be $1E5\mu/L^2$. Hence, the continuity eq. (1) is excluded from solution system and replaced by eq. (8).

5. Validation

5.1 Software validation

The grid dependency is checked together with continuity equation and obtained results showed an exactly validation of the velocity distribution for a grid size obtained by imposing an accuracy of 10^{-4}. This accuracy is a compromised value between the result accuracy and the time consumed in each run. The girded domain for Ri=1, Re=100 is shown in Figure (2a) and the distribution of $\left(\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}\right)$ over the domain is presented in Figure (2b).



(a) (b)

Figure (2). (a) grid distribution over the domain (b) validation of continuity equation .

5.2 Numerical results validation

A computational model is validated for mixed convection heat transfer by comparing the correlation of mixed convection in ventilated cavity with uniform heat flow in left wall performed by Rahman et al. [8]. Figure (3) shows the comperasion of the flow and thermal fields between the present investigation and Rahman et al.[8]. Figure (4) also shows another comparison of average dimensionless temperature as a function of Richarson numbers at Re=100, hi=0.2 and Pr=0.71. The results show a good agreement and from these comparisons it can be decided that the current code can be used to predict the flow field for the present problem.

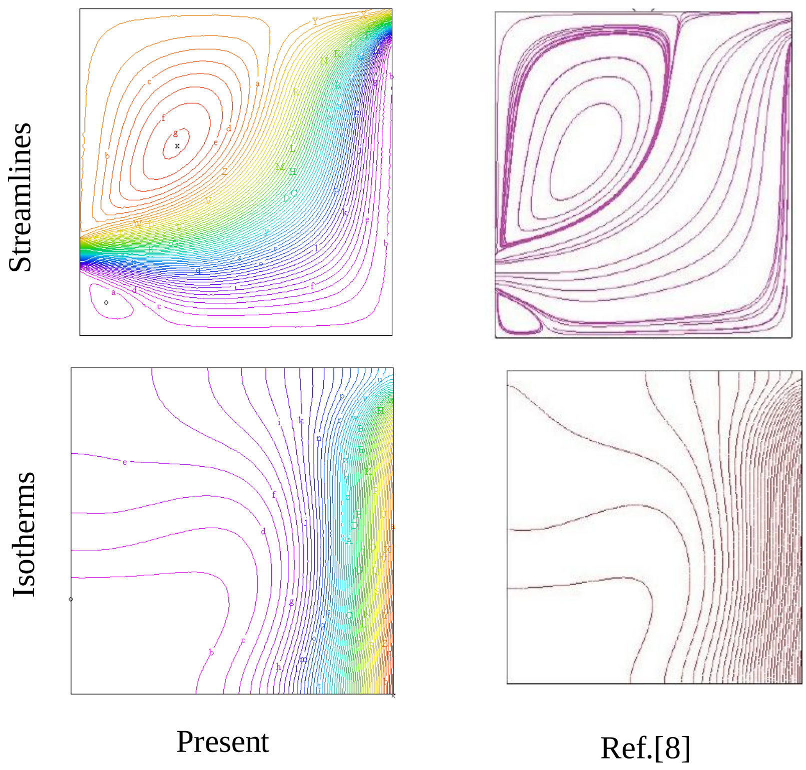


Figure (3). Comparison of streamlines and isotherms for validation at *Pr* = 0.71, *Re* = 100, *Ri* = 1.0 with the results of Ref.[8].

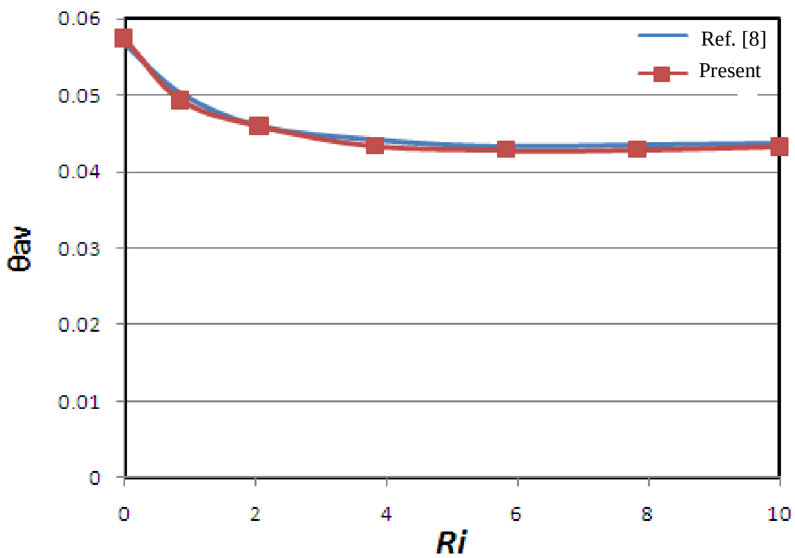


Figure (4). Comparison of average temperature as a function of Richardson numbers at for validation at *Pr* = 0.71, *Re* = 100 and hi=0.2 with the results of Ref.[8].

6. Results and discussion

A numerical study has been performed to analyze the laminar mixed convection heat transfer and air flow in a vented square cavity consist of a horizontal rod . Effect of the parameters such as Richardson number Ri, Reynolds number Re and the location of the inner cylinder on the heat transfer and fluid flow of the enclosure have analyzed.

6.1. Flow and temperature fields

Figure (5) show distribution of streamlines (on the left) and isotherms (on the right) for various locations of the inner heated cylinder at Ri=1 and Re=100.It can be seen from this figure that two circulating cells are formed, one is located at the top of the inlet port and the other cell at the bottom. The size of the upper cell is decreases when the inner cylinder location is moved from the lower part of the cavity e1=0.25 toward the upper part e3=0.75. Also figures show that for e1=0.25,the streamlines are arranged in parallel lines and cover all the surface of the cylinder while its cover the lower space of the cavity when the cylinder is moved toward the upper part of the enclosure. The formation of circulating cell is because of the mixing of the fluid due to buoyancy driven and convective current

Similarly the isotherms (on the right) are shown in the Figures (5a), (5b) and (5c) for different locations of inner heated cylinder e1=0.25, e2=0.5 and e3=0.75 respectively. For e1=0.25 (the heated cylinder is located at the bottom part of cavity) the wide heated region is concentrated at the space between the cylinder and the left vertical wall of the cavity. With moving the inner cylinder toward the upper, the isotherms are observed to have plum like distribution and occupy the upper part of the inner heated cylinder due to the growth of the thermal boundary layer. Because the effect of the location of inner heated cylinder, the isotherms show wavy variation and no vortices are observed contours.

Figure (6) shows the effect of Reynolds number on flow and isothermal fields at Ri=1 and the location of the inner cylinder at the center of the cavity (e2=0.5). It can be seen from Figure (6) (left) the intensity of the streamlines is increased with increasing of Re. Also with increases Re a bi-cellular vortex is seen just above and bottom the inlet port and occupies the left top and bottom space of the cavity. For isotherms this Figure show that with increasing of Reynolds number, the curvature of the isotherms lines is increased at the region which cover all the space between the heated walls due to dominant of forced convection. For a low value of Re a plum like distribution of isotherms is formed near the upper part of the heated cylinder and its direction is inclined to the right. As Re increases, the direction of the plum is changed to the left. This indicated that with low Re the effect of the buoyancy is increased leading to reducing the air density and induced the flow toward the exit.

Figure (7) shows the effect of Richardson number on the distribution of streamlines and isotherms at Re=100. Figure (7) (left) is depicted that the cellular motion just above the inlet port and the intensity of the stream lines are increased with increasing of Ri. It can be seen from this figure the plum like distribution of the hot isothermal lines is increased with increasing of Ri and cover most the space between the heated surfaces. The distribution of isotherms in the cavity at higher Ri is significantly different from that at lower values of Ri, because the buoyancy induced convection becomes more dominant.

6.2 Heat transfer characteristics

Plot of the average temperature (θ_{av}) of the fluid in cavity as a function of Re, at Ri=1 for different locations of inner cylinder is shown in Figure (8). It can be seen that the average temperature (θ_{av}) is decreased with increasing of Re due to the dominant of forced convection. It is clearly from this figure that the maximum values of average temperature of fluid is occurred when the inner cylinder is located at the center but when it moves toward the upper the values of the average temperature is decreased. This is because a significant effect of free convection when the inner cylinder is located at the center. This is because of a large heated area is cover most space of the cavity.

Figure (9) illustrate the variation of θav with Ri at Re=100. It has been observed that the average temperature of the fluid increases with increasing of Ri. When the cylinder moves closer to the upper cavity that transfers less than when it moves to the lower due to the secondary heat source that enhanced the natural convection at the lower part of the enclosure. The average Nusselt number at the hot wall of the enclosure as a function of Reynolds number for different locations of inner cylinder is shown in Figure (10). It observed that the value of Nusselt number increases with increasing of Re, also the $\overline{Nu}$ increases with moving the inner cylinder toward the upper location due to dominant of forced convection .

The effect of Richardson number on the average Nusselt number at heated vertical wall for different locations of inner cylinder at Re=100 is shown in Figure (11). For all locations of cylinder when Ri increases the average Nusselt number is gradually increases and the higher values of Nusselt number are observed for the upper location of heated cylinder. The behaviors results due to decrease of the average bulk fluid temperature when the heated cylinder in the upper location.

7. Conclusion

A numerical investigation is made of laminar mixed convection in a square cavity with concentric heated horizontal cylinder. The results are obtained for wide ranges of Richardson number (Ri) from 0 to 12 and range of Reynolds number (Re) from 50 to 200.

In view of the obtained results, the following findings have been summarized:

- The location of the solid cylinder has significant effect on the flow and thermal fields. The value of average Nusselt number is the highest in the mixed convection when the cylinder is located near the top wall of the cavity.

- The values of θ_{av} decreases with the increasing Re and with decreasing of Ri for different locations of the cylinder.

- The average Nusselt number at the heated surface is the highest for the highest value of Re and Ri.

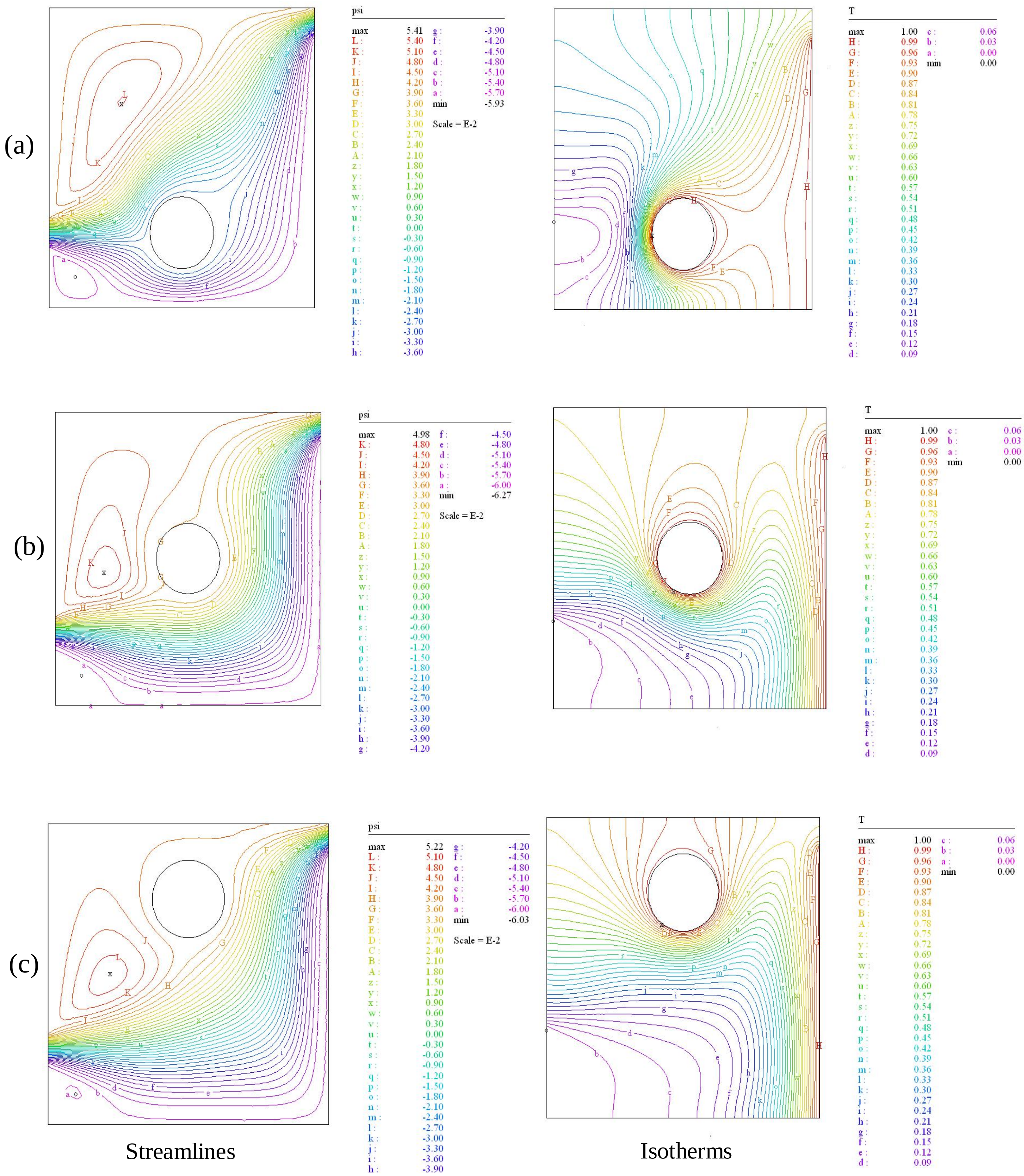
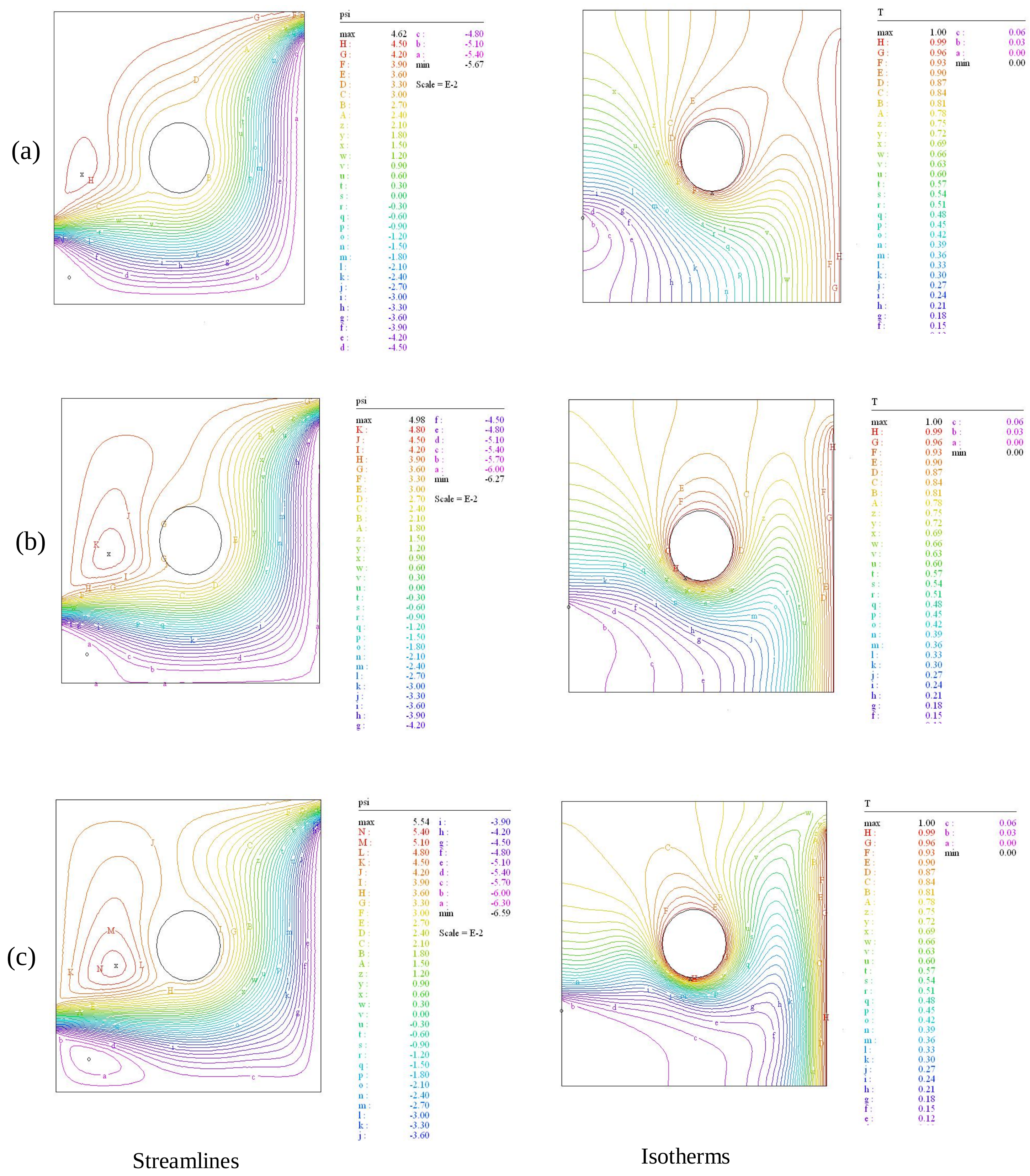


Figure (5). **Streamlines(left) and isotherms(right) at Ri=1and Re=100,(a)lower location(e1=0.25), (b)central location(e2=0.5), (c)upper location(e3=0.75) .**

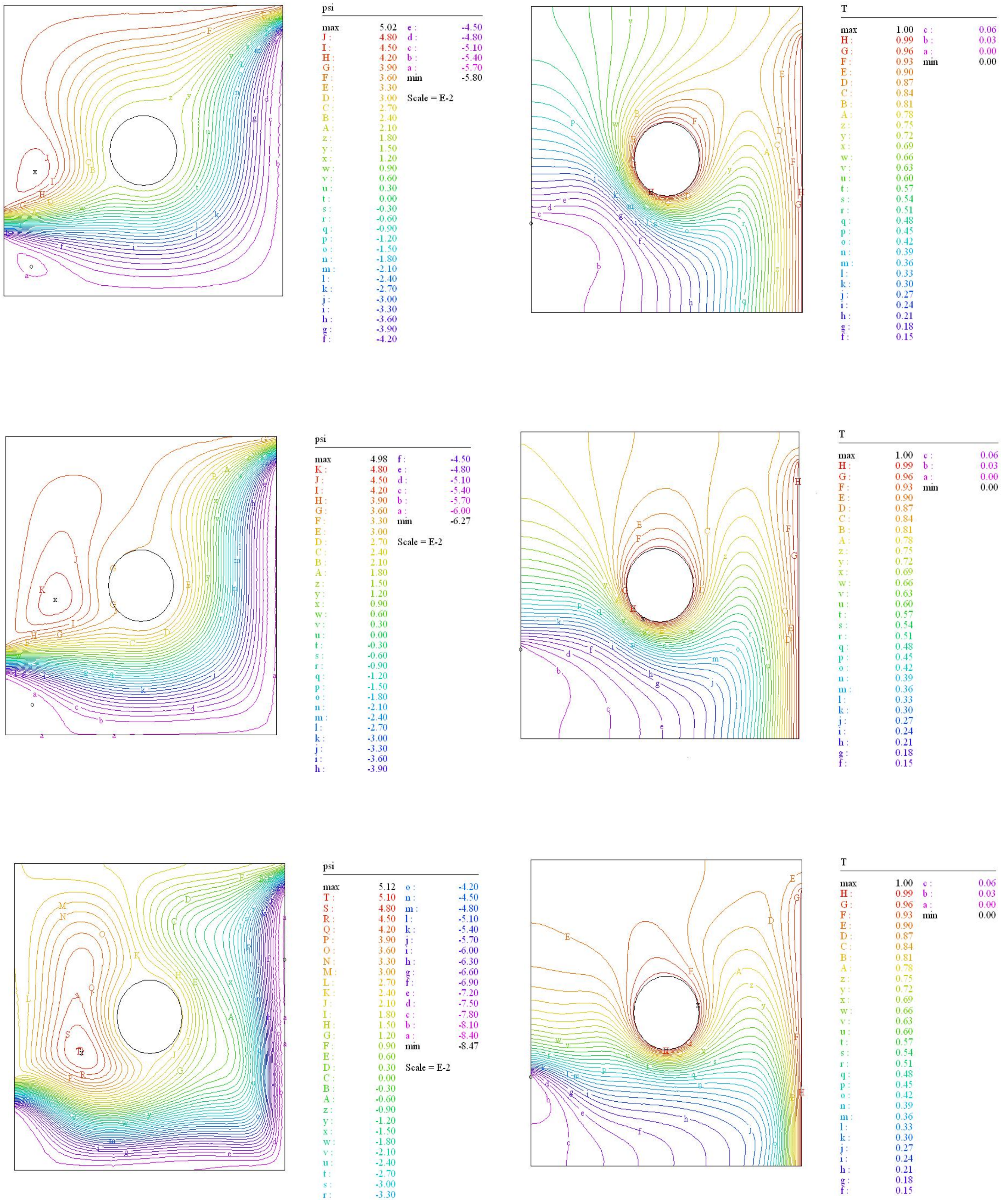


Figure(6). Streamlines (left) and isotherms (right) at Ri=1, (a) *Re* =50, (b) Re=100, (c) Re=200 .

(a)

(b)

(c)



Streamlines

Isotherms

Figure (7). Streamlines (left) and isotherms (right) at *Re* =100, (a) Ri=0, (b) Ri=1, (c) Ri=12.

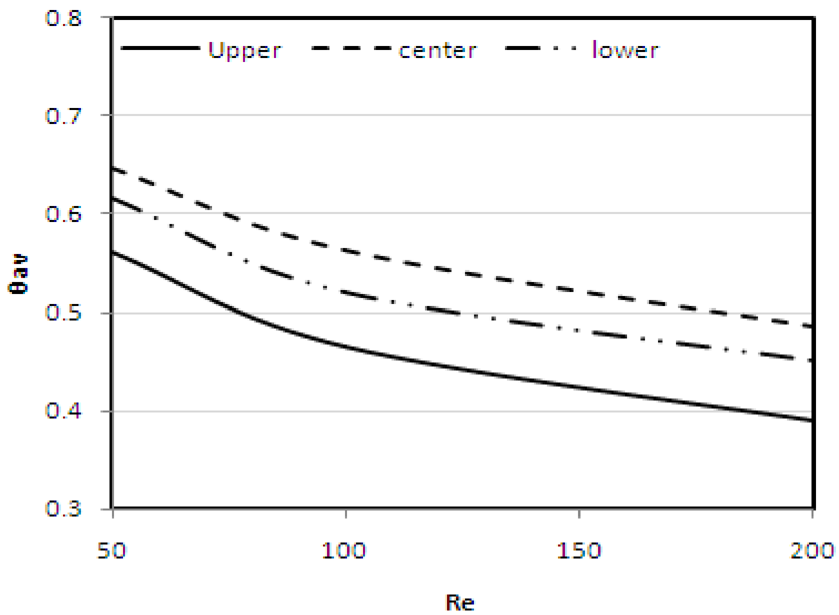


Figure (8). Variation of average fluid temperature with Reynolds numbers at Ri=1 for different location of inner cylinder .

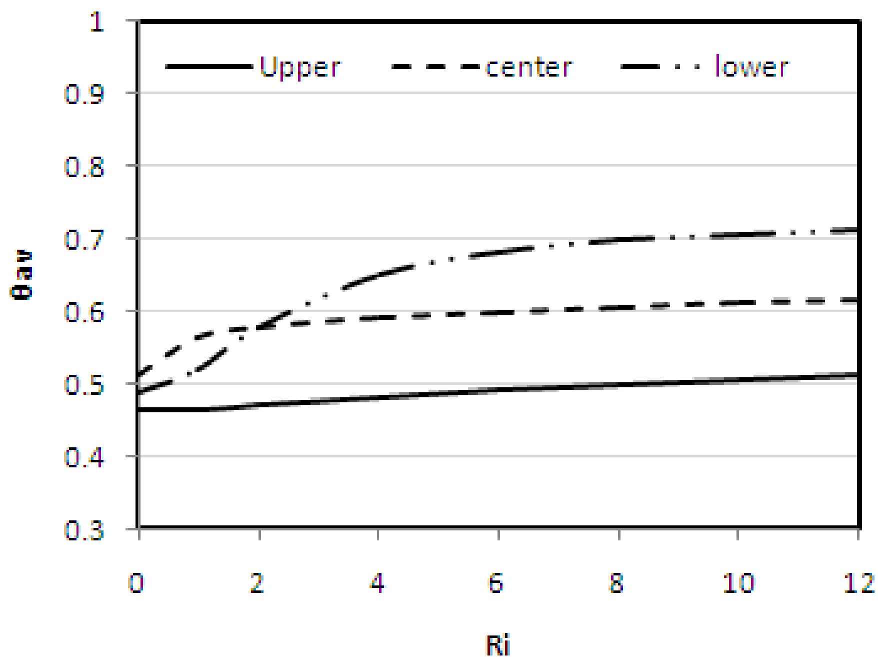


Figure (9). Variation of average fluid temperature with Richardson numbers at Re=100 for different location of inner cylinder.

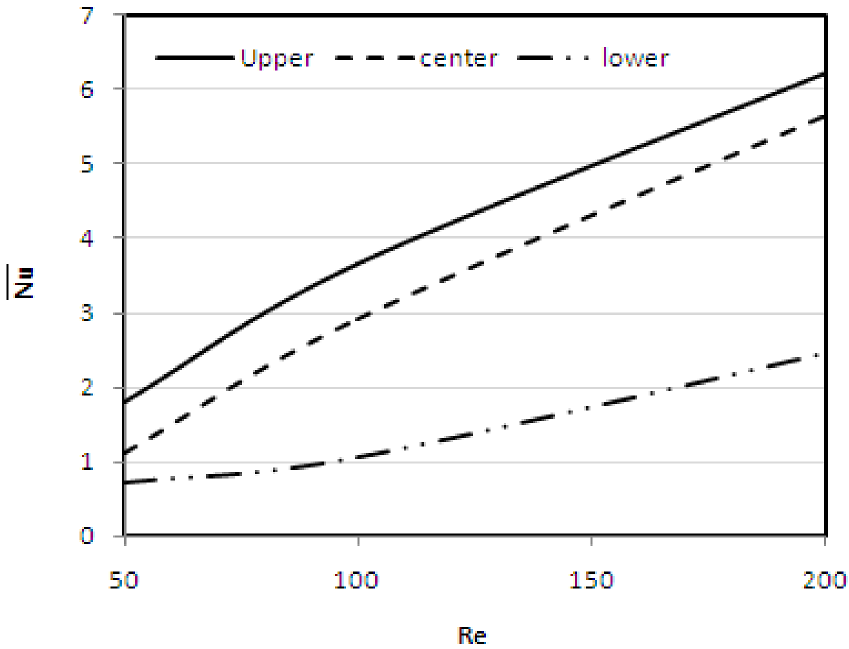


Figure (10). Variation of average Nusselt number with Re for different location of inner cylinder at Ri=1 at the heated wall .

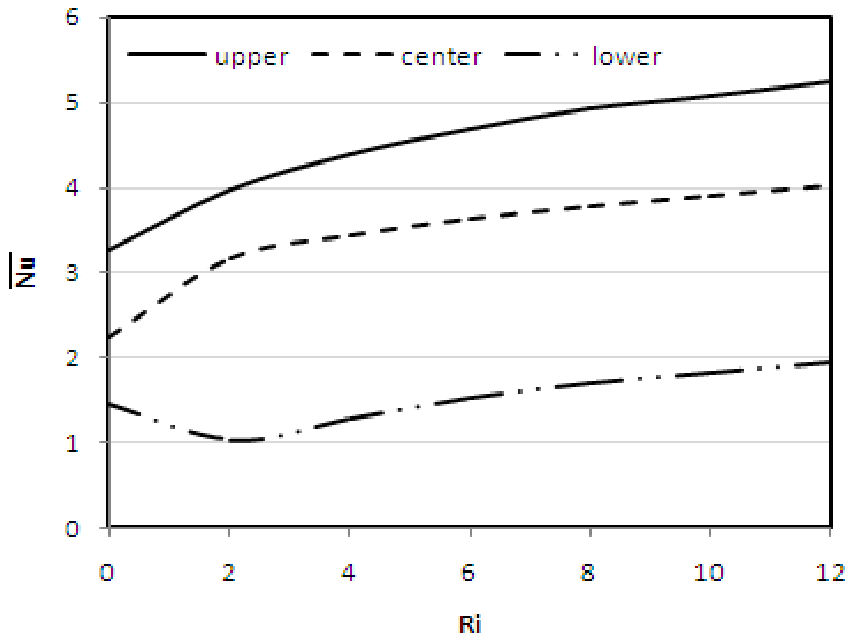


Figure (11). Variation of average Nusselt number with Ri for different location of inner cylinder at Re=100 at heated wall .

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9. Nomenclature

c_p	specific heat of the fluid at constant pressure ($Jkg^{-1}K^{-1}$)
g	gravitational acceleration (ms^{-2})
h_i	inlet port location
H_i	non-dimensional inlet port location
w	height of the inflow and outflow openings (m)
k	thermal conductivity of the fluid ($Wm^{-1}K^{-1}$)
L	length of the cavity (m)
L_s	length of the heated wall
Nu	Average Nusselt number
N	non-dimensional distance
p	pressure (N/m^2)
P	non-dimensional pressure, $p/\rho u_i^2$
Pr	Prandtl number, υ/α
Ra	Rayleigh number, $Gr\cdot Pr$
Re	Reynolds number, $u_i L/\upsilon$
Ri	Richardson number, Gr/Re^2
T	temperature (K)
T_h	Hot temperature (K)
T_i	Inlet temperature (K)
θ	non-dimensional temperature, $(T-T_i)/(T_h-T_i)$
θ_{av}	average non-dimensional temperature
u, v	velocity components (ms^{-1})
U, V	non-dimensional velocity components, u/u_i, v/u_i
V	cavity volume
x, y	Cartesian coordinates (m)
X, Y	non-dimensional Cartesian coordinates, x/L, y/L

Greek symbols

α	thermal diffusivity, $k/\rho Cp$ (m^2s^{-1})
β	thermal expansion coefficient (K^{-1})
ρ	density of the fluid (kgm^{-3})
υ	kinematic viscosity of the fluid (m^2s–1)

Subscripts

av	average
s	heated surface
i	inlet state