Estimation of Flow in Horizontal Elliptic Channels with Free Overfall at the End of the Channel

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Abstract

The aim of this work is to calculate the flow discharge in horizontal elliptic channels with free overfall at the end of the channel. Two methods are used, the first one is Boussinesq approximation to calculate the End- Depth – Ratio (EDR) .The second one is a theoretical procedure applied to compute the flow over a sharp – crested weir.The available experimental data are used to verify the proposed End – Depth – Discharge (EDD) relationship. The calculated discharges, using the proposed EDD relationship, show excellent agreement with the experimental values in subcritical flows. However, the agreement is not so good in supercritical.

Keywords : Open channel flow ; Free overfall ; End – depth.

المستخلص

الهدف من هذا البحث هو حساب كمية التصريف في القنوات ذات الشكل البيضوي من خلال الطفح الحاصل في نهاية القناة .استخدمت لهذا الغرض طريقتان, الطريقة الاولى تمت باستخدام تقريب (Boussineq) لحساب نسبة العمق النهائي الى العمق الحرج. اما الطريقة الاخرى فهي حساب كمية التصريف فوق السد المنحي الحاد في نهاية القناة مع فرض ضغط الماء مساوي صفر عند تلك النهاية . من خلال الاستعانة بالنتائج العملية لدراسات سابقة ومقارنتها مع النتائج النظرية لقيم التصريف وجد ان قيمة التصريف المحسوبة من خلال نسبة العمق النهائي الى التصريف متوافقة تماما مع القيم العملية في حالة الجريان شبة الحرج , بينما ظهر عدم التوافق للنتائج في الحالتين سابقتي الذكر في حالة الجريان في الوضح الحرج جدا".

1. Introduction

A free overfall occurs when the flow detaches from the solid boundary to form a free nappe owing to an abrupt decrease in channel bed elevation (that is a drop structure)[1]. It offers a method of discharge measurement in open channels from a single measurement of the depth at the brink known as end depth . The value of the end depth depends on the shape of the approach channel. If the slope of the channel is negative, zero or mild, the flow at upstream of the fall will be critical. However, if the upstream channel is steep, the flow will be supercritical and normal depth occurs upstream of the brink[2].

The measurement of flow discharge in open channels is useful especially in channels having covers (sewer, duct, tunnel etc.) . Rouse [3], being the first to investigate the end-depth experimentally, proposed a relationship termed End-Depth Ratio (EDR = end-depth/critical-depth), which was found to be 0.715 in mildly sloping rectangular channels.Diskin [4] applied a momentum equation between apparent and end sections and obtained an expression for end-depth ratio (EDR), i.e., ratio of end depth to critical depth, *YC*. Rajaratnam and Muralidhar [5] introduced a pressure coefficient in the momentum equation and calculated EDR = 0.667, 0.731 and 0.775 for subcritical flows in rectangular, parabolic and triangular channels, respectively. Dey [6] extended the use of the momentum equation to calculate EDR for others shapes of channels. The generalized energy method gives EDR = 0.694, 0.734 and 0.762 for subcritical flows in rectangular, parabolic and triangular channels, respectively. Anastasiadou-Partheniou and Hatzigiannakis [7], Ferro [8] and Ahmad [2] simulated the free overfall with a sharp crested weir of zero height. Marchi [9] solved the two dimensional free overfall using cnoidal wave theory.

Dey [10] presented a theoretical and experimental study on free overfall in an inverted semicircular channel. He applied the momentum equation between the apparent section and the end section and found that EDR = 0.705 for subcritical flows up to the ratio of critical depth, and diameter 0.42. However, in the supercritical flow, the EDR decreases with increase in relative bed slope (ratio of critical bed slope, S_C, to bed slope, S). He also found that computed discharges obtained through application of a momentum equation in supercritical flows are not comparable with the experimental ones. The reason for the disagreement is due to neglecting the stream-wise component of the gravity force in the momentum equation.

In this paper, two separate methods are presented to analyze the free overfall in elliptic channels, as shown in Figure (1)(a, b and c). First, an analytical model for a free overfall from smooth elliptic channels is presented, applying a momentum approach based on the Boussinesq assumption. Secondly, an alternate approach for a free overfall from elliptic channels is also presented. The model yields the end-depth ratio and end depth– discharge relationship, which are verified using the experimental data of previous studies.

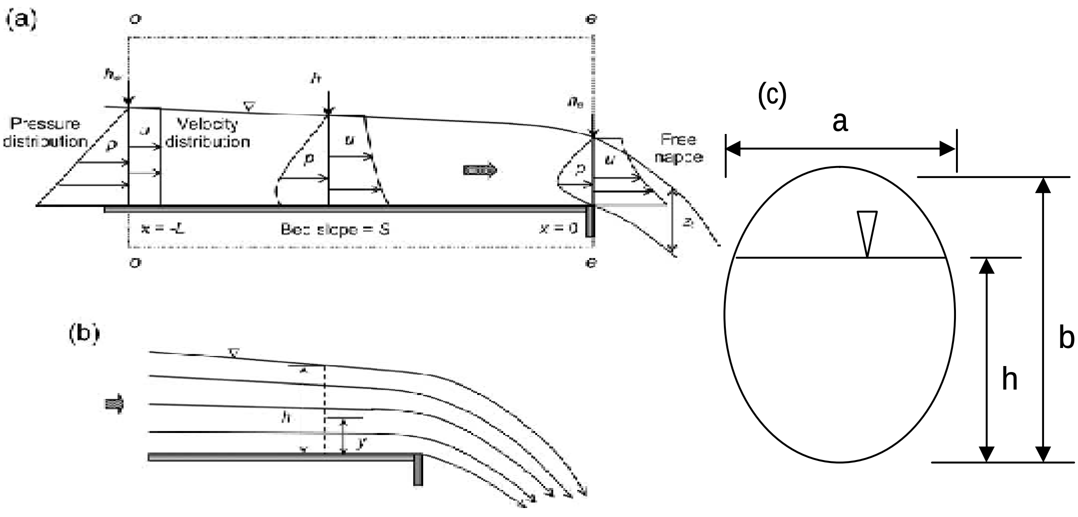

Figure (1). (a) Schematic view of a typical free overfall.

(b) Streamline pattern of a free overfall.

(c) Cross –section of an elliptic channel.

2. Problem Formulation

2.1. Boussinesq approximation

The free surface curvature of a free overfall being relatively small varies from a finite value at the free surface to zero at the channel bed as shown in Figure (1). According to the Boussinesq approximation [11], a linear variation of the streamline curvature with depth y is assumed. Hence, the radius of curvature r of a streamline at y is expressed by:

$$1/r = (y/h)(1/r_s) \tag{1}$$

Where

h =flow depth, and r_s=radius of curvature of the free surface. For the small free surface curvature, it can be approximated by:

$$1/r_s = d^2h/dx^2 \tag{2}$$

Where:

x = streamwise distance. The normal acceleration ay based on the aforementioned assumption is given by:

$$a\,y = k\,y \tag{3}$$

Where:

$$K = \frac{V^2}{h}\frac{d^2 h}{dx^2} \tag{4}$$

Where V = mean flow velocity. Integrating the Euler equation (see Subramanya [12]), the effective hydrostatic pressure head h_{ep} is expressed by

$$h_{ep} = h + \frac{kh^2}{3g} \tag{5}$$

Where g =gravitational constant. Eq. (5) obtained by Boussinesq [11] is applicable in solving problems with small curvature at the free surface, namely free overfall**.**

The flow velocity at the end section is calculated by applying the Bernoulli equation on a streamline between the upstream section at x =- L and the end section at x = 0. The discharge Q is computed using the following equation:

$$Q = Cd\sqrt{2g}\int_0^{h_0} t\,\sqrt{(H-y)dy} \tag{6}$$

Where Cd = coefficient of discharge (= Cv Cc), Cv= coefficient of velocity, Cc = coefficient of contraction, t =channel width at an elevation y, and H = total head. The total head H at the upstream section (x = -L) is given by

$$H = h_0 + \frac{V_0^2}{2g} \tag{7}$$

2.2. End-depth–discharge relationship

The free overfall is a special case of a sharp-crested weir, at the end of the channel and of zero height. The theoretical procedure applied to compute the discharge over a weir can also be applied to a free overfall to get the EDD relationship [8]. The flow velocity at any depth at free overfall is calculated by applying the energy equation between any two points located at sections o–o and e–e (see Figure (1).(a)). It is assumed that all the streamlines at the brink are parallel to each other, i.e., the emerging jet is undeflected. To account for the curvature of streamlines, i.e., the deflection of the jet due to gravity, a coefficient of contraction is considered. Zero pressure at the brink is assumed. The discharge through an elemental strip of thickness dyat a height y above the bed is given by:

$$dQ = 2g\{H - y\}^{1/2} T\, dy \tag{8}$$

Where g = acceleration due to gravity; $\boldsymbol{H = Y_1 + \frac{V_1^2}{2g}}$, T = top width, given by the following relationship:

$$T = (D_2 - 4y_2)^{1/2} \tag{9}$$

Where D = diameter of the channel. Substituting Eq. (9) into Eq. (8), the total discharge is given by

$$Q = C_C\sqrt{2g}\int_0^{Y1} \frac{(H - y_1)}{2D}\left(1 - \frac{y_2}{D_2}\right)^{1/2} dy \tag{10}$$

Where C_C= coefficient of contraction. Rearranging Eq. (10), one can get

$$F_1 = \frac{V_1}{\sqrt{2gA_1/T_1}} \tag{11}$$

$$A_1 = \frac{D^2}{8}\left[2sin^{-1}(2X_1) + 4X_1\sqrt{(1 - 4X_1^2)}\,\right]$$

$$= \frac{D^2}{8}\emptyset_1(X_1) \tag{12}$$

$$T_1 = D\sqrt{(1 - 4X_1^2)} = D\emptyset_2(X_1) \tag{13}$$

Where:

$$X_1 = \frac{Y_1}{D} \tag{14}$$

$$H = Y_1\left(1 + \frac{F_1^2 A_1}{2Y_1T_1}\right) \tag{15}$$

Substitution of Eqs. (12) and (13) into Eq. (15) yields

$$\frac{H}{D} = X_1 \left[1 + \frac{F_1^2}{16X_1} \frac{\emptyset_1(X_1)}{\emptyset_2\,(X)_2} \right] \quad (16)$$

Ahmad [13] theoretically solved the EDR in circular channels, as was done by Dey [6], who solved equations numerically for the EDR, where Ahmad [13] used a series to solve the equations. He derived the following EDD relationship for subcritical flow :

$$Q^{\wedge} = \frac{Q}{g^{1/2}D^{5/2}} = \frac{1}{16\sqrt{2}} \frac{\emptyset_1\,(X_C)^{3/2}}{\emptyset_2(X_C)^{1/2}} \quad (17)$$

2.3. Theoretical approach

The set of characteristic parameters appropriate for free overfall phenomenon at the end of a channel can be given in functional form as follows:

$$Q = f_1\,(h_e, l, m, g, \mathrm{m}) \quad (18)$$

where l = characteristic length parameter of a channel, and m = dynamic viscosity of fluid. Using the Buckingham p-theorem and selecting the parameters l, g and m as repeating variables, the non dimensional parametric equation in functional form can be given by:

$$Q^{\wedge} = f_2(h_e\hat{}, m) \quad (19)$$

Where $Q^{\wedge} = Q/\,(g^{0.5}\ l^{\,2.5})$, and $h\hat{}\,e = he\,/\,l$. As it is not appropriate to use m as a free parameter, a refinement of the above equation can be done as:

$$Q\hat{} = f_3\,(h_e\hat{}\,) \quad (20)$$

Dey [13] theoretically analyzed free overfall in horizontal elliptic channels, using the momentum equation based on the Boussinesq approximation. So, we can use equation (17) to estimate the EDD for elliptic channel with free overfall by substituting the major axis (a) , with the diameter of channel (d) as shown below :

$$Q^{\wedge} = \frac{Q}{g^{0.5}\ .\,a^{2.5}} \quad (21)$$

respectively, for different values of λ(=a/b), where a is the major axis, and b is the minor axis, as shown in Figure (1.c).

3. Calculation of Flow Discharge in Horizontal Elliptic Channel With Free Over fall

First, the free overfall from elliptic channels has been calculated by applying the momentum equation based on the Boussinesq approximation. The method eliminates the need of an experimentally determined pressure coefficient. In subcritical flows, the EDR has been related to the critical-depth. On the other hand, in supercritical flows, the end-depth has been expressed as a function of the streamwise slope of the channel using the Manning equation. The mathematical solutions allow estimation of discharge from the known end-depth in subcritical and supercritical flows. Streamline curvature at the free surface has been used to compute the upstream flow profiles of a free overfall. The comparisons of the experimental data with this model have been satisfactory for subcritical flows and acceptable for supercritical flows.

The discharge of an elliptic channel with 4 m long and λ (=a/b) ranging from 0.3 to 4 was calculated by using the following equation which was derived by Rohwer [15] :

$$Q = 8.58\ a^{0.62}\ .\ b^{1.82} \qquad (22)$$

While The EDD ($Q^{\wedge}$) is calculated from equation (21) for an elliptic channel with EDR ($h_e^{\sim}$) $= h_e / h_c$, which can be calculated for two cases:

3.1 Subcritical Flow

The EDR predicated from momentum equation is:

$$h_e^{\sim} = \left(\frac{2F_1^2}{1 + 2F_1^2}\right)^{\frac{2}{3}} \qquad (23)$$

Where $$F_1 = \frac{Q}{(g\ d\ h_0^4)^{0.5}} \qquad (24)$$

3.2 Supercritical Flow

In supercritical flow, the EDR $h_e^{\sim}$ is expressed as a function of relative slope S and $h_c^{\wedge}$ using Manning equation . Figure (2) show the relationship between EDR ($h_e^{\sim}$) and $h_c^{\wedge} = (h_c / a)$, Figure (3) present the variations of $h^{\wedge}_e$ $(=h_e/a)$ with $Q^{\wedge}$.

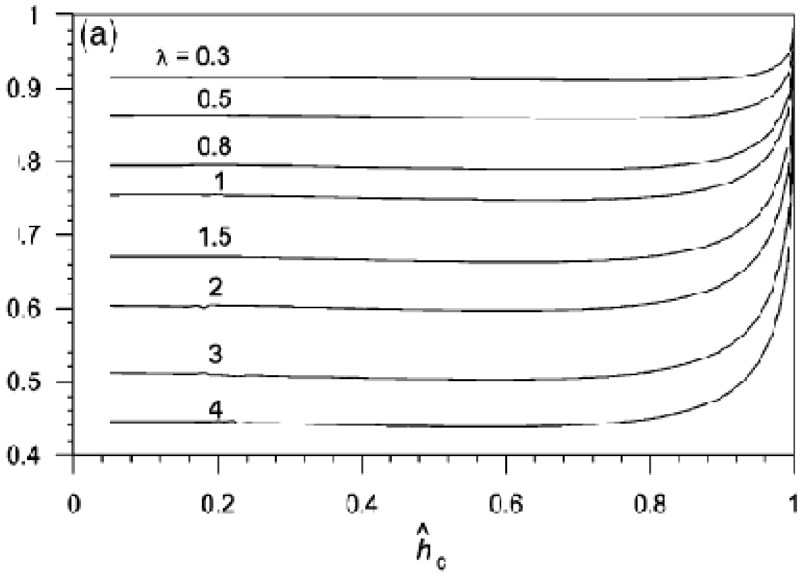

Figure (2). Dependency of EDR $h_e^{\sim}$ on $h_c^{\wedge}$ for different l.

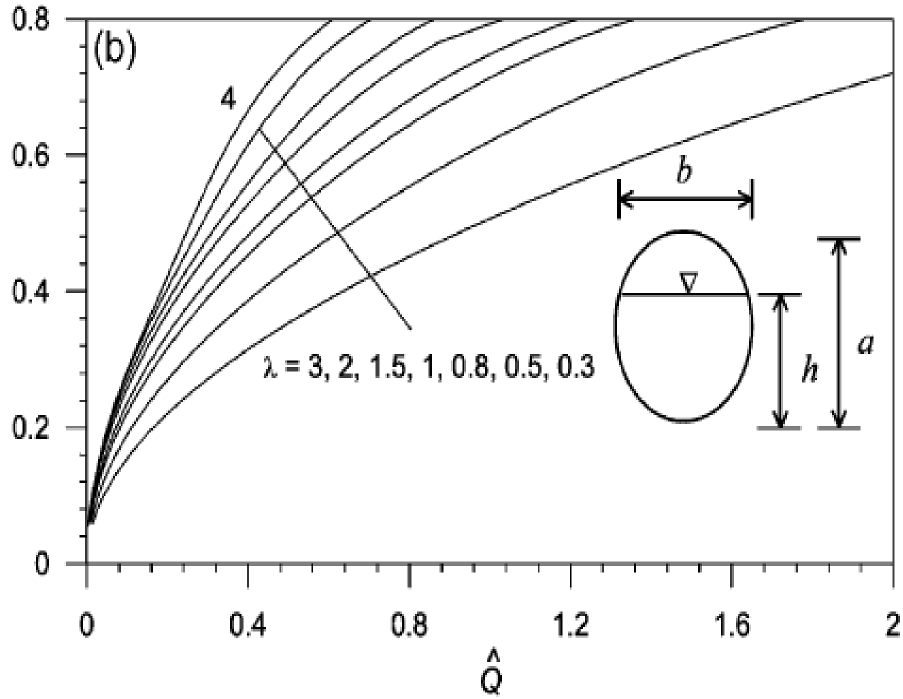

Figure (3). Variation of $h_e^{\wedge}$ with $Q^{\wedge}$ for different l in elliptic free over fall.

4. Results and discussion

From the computations of flow discharge in the end of the channel and comparative the variations of EDR $\tilde{h}_e$ with $\hat{h}_c$ as shown in Figure (2) , we can notice that the value of EDR $\tilde{h}_e$ is approximately 0.725 for a wide range of $\hat{h}_c$. Figure (3) show that The EDD $\hat{Q}$ increases with increase in $\hat{h}_e$. Table (1) shows that the computed values of $\hat{Q}$ have a slight variation from the experimental observations of Dey [13] due to the assumption of pseudo-uniform flow and the use of the Manning equation in small channels. The calculated discharges, using the proposed EDD relationship, show excellent agreement with the experimental values in subcritical flows. However, the agreement is not so good in supercritical.

Table (1). Comparisons of experimental data of Dey [1] with computational data obtained from present analysis.

End – Depth - Ratio			Experimental	Computed
$\hat{h}_c$	$\hat{h}_e$	$\tilde{h}_e$	$\hat{Q}$	$\hat{Q}$
0.206	0.149	0.721	0.0865	0.0917
0.2462	0.175	0.711	0.1138	0.113
0.45	0.325	0.721	0.2984	0.340
0.478	0.345	0.723	0.3296	0.367
0.529	0.38	0.719	0.3911	0.412
0.497	0.357	0.718	0.3477	0.380
0.323	0.232	0.719	0.1753	0.227
0.361	0.261	0.726	0.2092	0.262
0.169	0.124	0.736	0.0673	0.075
0.193	0.144	0.749	0.0825	0.090

5. References

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6. Nomenclature

a	Major axis of elliptic channel (m)
A	Flow area (m^2)
b	Minor axis of elliptic channel (m)
C_c	Coefficient of contraction
C_d	Coefficient of discharge
C_v	Coefficient of velocity
d	Channel diameter (m)
E	Specific energy (m)
F	Froude number of approaching flow
g	Gravitational constant ($m.s^{-2}$)
h	Flow depth (m)
$\hat{h}$	h/a
$\tilde{h}$	h/h_c
h_{ep}	Effective hydrostatic pressure head (m)
H	Total head (m)
l	Characteristic length of channel (m)
L	Length of control section (m)
Q	Discharge ($m^3.s^{-1}$)
$\tilde{Q}$	Free over low of elliptic $Q/(g^{0.5} . a^{2.5})$
S	Streamwise slope
T	Top width of flow (m)
V	Mean velocity of flow ($m.s^{-1}$)
λ	= a/ b

Subscripts

c	Critical flow
e	End section
o	Upstream section