

## Active Vibration Control of Linear and Nonlinear Quarter Car Suspension System Using PID and Fuzzy Logic Controllers

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### **Abstract**

A quarter car suspension linear and nonlinear models are considered for the investigation of active vibration control using PID (Proportional-Integral-Derivative) and Fuzzy logic controllers. Simulation of the models using matlab-simulink have been built in this work for passive and active vibration control. The Fuzzy rules and memberships were taken from the literature based on practical experience. A comparison between the linear and nonlinear models using different type of control methods have been carried out. Results shows that the Fuzzy-PID control method is the best among other methods, and the nonlinear model gives better results than the linear model for active control.

**Key words:** Active, Vibration, Control, Car Suspension, Linear, Non-Linear, PID, Fuzzy Logic

**السيطرة الفعالة على الاهتزاز لربع منظومة تعليق سيارة خطية وغير  
خطية باستخدام مسيطرات (بي آي دي) والمنطق الضبابي**

### **الملخص**

نماذج خطية وغير خطية لربع منظومة تعليق سيارة تم اعتبارها لغرض التحري عن سيطرة الاهتزاز الفعالة باستخدام مسيطرات (بي آي دي) والمنطق الضبابي. محاكاة للنماذج باستخدام برنامج الماتلاب بنيت في هذا العمل لغرض السيطرة على للاهتزاز الاعتيادية والفعالة. القواعد الضبابية والعلاقات تم اخذها من البحوث اعتمادا على الخبرة العملية. مقارنة بين النماذج الخطية والاختية باستخدام طرق سيطرة مختلفة تم اجراءها. النتائج بينت ان طريقة السيطرة

المسماة (Fuzzy-PID) (بي أي دي- الضبابية) هي الافضل من بين الطرق الاخرى، وان النموذج الغير خطي اعطى نتائج افضل من النموذج الخطي للسيطرة الفعالة.  
**كلمات مرشدة :** فعالة، اهتزاز، سيطرة، منظومة تعليق سيارة ،خطية، غير خطية، تناظري ،تكاملي،تفاضلي،المنطق الضبابي

### 1-Introduction

Modeling and control of car suspension system are an essential part of automotive engineering. Passive suspension systems are widely used in passenger vehicles but cannot effectively suppress vehicle vibrations under different road conditions. Active controlled suspension systems are more reliable and efficient than other methods. The success of vibration control for improving the ride comfort is determined by two stages. In particular, the accurate modeling and designing of an appropriate controller have great importance in reduction of vehicle vibrations [1]. Car suspension systems are often modeled as linear characteristic for mathematical simplicity. Nevertheless, real suspension system behavior is nonlinear. In order to account for the nonlinear behavior of car suspension systems, several studies have been found in the literature. Duyn [2] developed a nonlinear damper model , and Yung et al. [3] validated this model for frequencies up to 500 Hz. Dry friction forces that arise from damper have been also studied in the literature. Stein et al. [4] focused on the dry friction modeling and simulation of single degree of freedom.

vibration systems. Also Guclu [5] investigate the the dynamic behavior of a car model which includes the nonlinearities that arise from the dry friction on the dampers. Yagiz et al. [6] presented a nonlinear model of half car system which includes a nonlinear spring and a piecewise linear damper with dry friction. Saban and Ozgur [7] presented a two degree of freedom nonlinear quarter car model. The nonlinearities that were studied in the paper were cubic stiffness, nonlinear damping, and coulomb friction damping in the suspension.

Controlling active suspension systems have considerable effect on the ride comfortability. Control techniques such as PID, Fuzzy, and Fuzzy-PID are employed to increase the overall performance of suspension systems by adjusting the actuator force. Several studies published in the literature related with the control of nonlinear car suspension models such as References [1,5,6,7]. Salem and Aly [8] illustrated the application of fuzzy logic technique to the control of quarter car suspension model. The paper also shows a comparison of active suspension fuzzy control and PID control

using Matlab. Yaujung et al. [9] focuses on the analysis and control of a semi-active suspension system for light vehicles. Models of a quarter-car suspension with air spring and magneto rheological damper were built. Because components in the system involve nonlinear dynamic characteristics, a self-tuning Fuzzy logic controller was designed to the semi-active suspension system. The simulation results showed that the designed suspension system and its controller had good performance for vibration suppression. Pekgokgoz et al. [10], used the fuzzy logic and proportional, integral, derivative (PID) controller methods, the vehicle body deflections and the control force have been obtained and compared with each other. The comparisons displayed the efficiency and convenience of the fuzzy logic controller (FLC) method. Goegoes et al. [11] presented state feedback controller of a quarter-car active suspension system. The controller structure of the active suspension system was decomposed into two loops namely outer loop and inner loop controllers. Outer loop controller is used to calculate the optimum target force to eliminate the effect of road disturbances, while, the inner loop controller is used to keep the actual force close to this desired force. The performance of outer loop controller also shows significant improvement in terms of body acceleration, displacement and suspension displacement as compared to the passive suspension system. Nemat et al. [12], illustrated the application of fuzzy logic technique to the control of continuously damping car suspension system. The paper also described the model and controller used in the study and discussed the response obtained from a range of different input simulations. Choudhury et al. [13] studied the performance of two basic suspension systems with a different approach, passive and active suspension system. Quarter car model was used for simulation of the system. To analyze the model Matlab/Simulink was used and the results show that active suspension system has better ability to reduce the disturbance and also provides better damping quality than passive suspension system. Sawant et al. [14] presented a comparison between simulation results obtained for passive and semi-active linear systems with nonlinear mass, spring, and damper controller. The excitation was taken as actual random road excitation to achieve improved performance. Ebrahimi et al. [15] presented an optimal fuzzy controller developed to improve the performance of active suspension system. Fuzzy logic was used to tune each parameter of the PID controller. They also show that the performance of active suspension system has improved significantly compare to conventional PID controller. Abdolvahab et al. [16] obtained a mathematical model for the passive and active suspension systems for

quarter car model subjected to excitation from road profile. The active suspension system was synthesized by a Linear Quadratic Regulator (LQR) control technique for a quarter car model. The performance of the controller was determined by computer simulation using Matlab/Simulink toolbox.

In this study, simulations and comparisons of different control methods for linear and nonlinear quarter car suspension systems will be carried out using Matlab/Simulink, since there was no such a study or investigation were found in the literature.

## **2- Mathematical Model of Quarter Nonlinear Car Suspension System**

Car suspension systems are often modeled as ideal linear systems despite that real suspension systems have nonlinear characteristics. The first step on setting up a mathematical model for vibration control of a car suspension is to define the nonlinearities in the system. The analysis of nonlinearities in car suspension system is well documented in the literature. A two degree of freedom nonlinear quarter car suspension model, given in reference [7], which is shown in Fig. 1 is considered for this study.

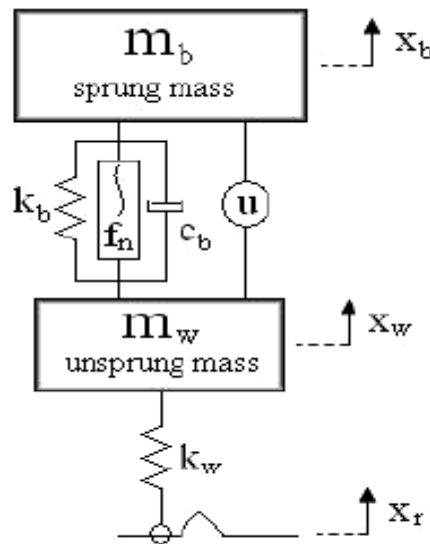


Fig. 1 Nonlinear quarter-car model

The model consists of a quarter car body and a wheel which are connected with a suspension system. The suspension system is consisted of spring, damper, and

nonlinear elements. The nonlinearities considered in this model are cubic stiffness, nonlinear damping, and coulomb friction damping in the suspension. An actuator is also connected between the car body and the wheel, and the wheel spring is in contact with the road profile. The dynamical differential equations of this model can be written as follows:

$$m_b \cdot \ddot{x}_b + c_b(\dot{x}_b - \dot{x}_w) + k_b(x_b - x_w) + c_b^n(\dot{x}_b - \dot{x}_w) \cdot \text{sgn}(\dot{x}_b - \dot{x}_w) + k_b^n(x_b - x_w) + \mu \cdot f_{susp} \cdot \text{Sgn}(\dot{x}_b - \dot{x}_w) = u \quad (1)$$

$$m_w \cdot \ddot{x}_w - c_b(\dot{x}_b - \dot{x}_w) - k_b(x_b - x_w) - c_b^n(\dot{x}_b - \dot{x}_w) - k_b^n(x_b - x_w) + k_w(x_w - x_r) - \mu \cdot f_{susp} \cdot \text{Sgn}(\dot{x}_b - \dot{x}_w) = -u$$

where  $m_b$  represent 1/4 car body mass, and  $m_w$  is the wheel mass.  $k_b$  and  $k_w$  are the linear stiffness of the suspension car body and the car tire respectively.  $c_b$  is the linear damping constant,  $k_b^n$  and  $c_b^n$  are the nonlinear elements of the suspension system.  $\mu$  is the friction coefficient and the suspension normal force for Coulomb damping is  $f_{susp} = 0.1m_b \cdot g$  [17].  $x_b$ ,  $x_w$ , and  $x_r$  are the vertical displacements of the car body, wheel displacement, and road profile displacement respectively, and  $u$  is the actuator control force.

### 3- Control Methods of Quarter Car Suspension Systems

The aim of control of car suspension systems is to minimize the vibration level of vehicle body as much as possible. Classical PID controllers are widely used and appropriate for car suspension control. Nevertheless, because of its linear characteristics, classical PID controllers are not preferred to control nonlinear systems[1]. In order to cope with nonlinearities a fuzzy logic controllers (FLC) is used in this study to reduce the vibration level of the car body. In addition, the hybrid-type control combining fuzzy logic and PID is also investigated.

#### 3.1 Classical PID Controller

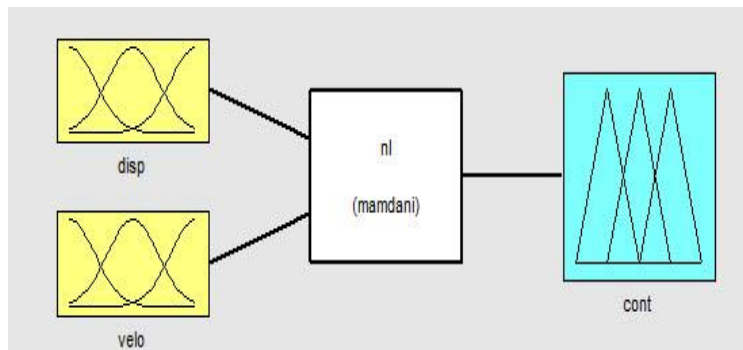
The Conventional PID controllers which combine Proportional, Integral, and Derivative actions linearly are most common controllers applied in industry. The typical PID control law is given by

$$U(t) = K_p \cdot e(t) + K_I \cdot \int_0^t e(\tau) d\tau + K_D \cdot \dot{e}(t)$$

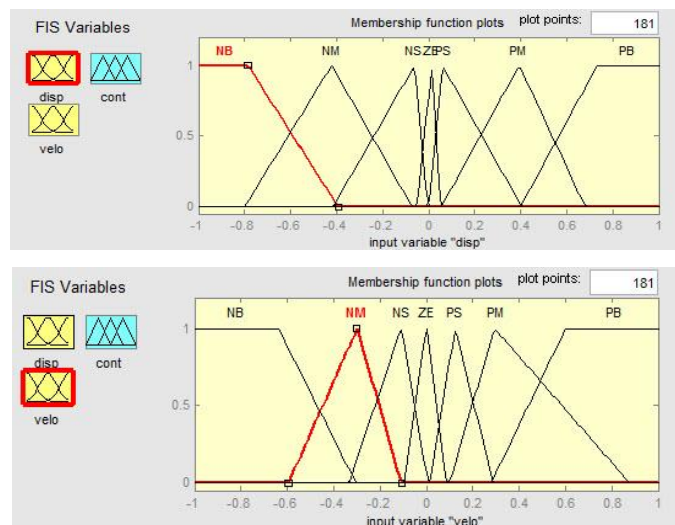
where  $e(t)$  is the error defined as  $e(t) = x_{ref} - x_b$ .  $U(t)$  is a control variable,  $K_P$  is a proportional gain,  $K_I$  is an integral gain, and  $K_D$  is a derivative gain.

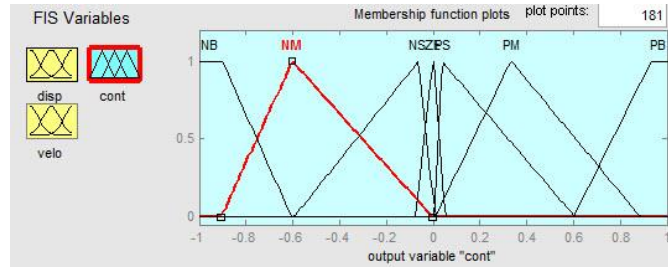
### 3.2 Fuzzy Logic Controller

Fuzzy logic has been widely used for active control problems. Its theory based on linguistic synthesis and does not need an exact mathematical model while designing the control system. Thus, the controller performance is independent from the accuracy of the mathematical model. The classical fuzzy logic controller based on an expert's knowledge or experience has two inputs and one output. The error ( $e$ ) and change of the error ( $de$ ) are used for producing a control output ( $u$ ). In this work the error is taken as the relative displacement ( $x_b-x_w$ ) and the change of error is the relative velocity ( $x_b^{\circ}-x_w^{\circ}$ ). Using the matlab fuzzy logic toolbox shown below:



The abbreviation correspond to linguistic variables have been classified as: **NB** (Negative Big), **NM** (Negative Medium), **NS** (Negative Small), **ZE** (Zero), **PS** (Positive Small), **PM** (Positive Medium), and **PB** (Positive Big). Trapeze and triangular membership functions are utilized. All the member ship functions of input and output functions are depicted below:





A rules based on fundamental knowledge and experience of the system and cover the input-output relations that define the fuzzy control strategy are given in reference [7] in table form as follows:

TABLE I  
EXAMPLE OF DECISION TABLE

de/e	NB	NM	NS	Z	PS	PM	PB
NB	NB	NB	NB	NM	NM	NS	ZE
NM	NB	NB	NM	NS	NS	ZE	PS
NS	NB	NM	NS	NS	ZE	PS	PM
Z	NM	NS	NS	ZE	PS	PS	PM
PS	NM	NS	ZE	PS	PS	PM	PB
PM	NS	ZE	PS	PS	PM	PB	PB
PB	ZE	PS	PM	PM	PB	PB	PB

### 3.3 Fuzzy Logic PID Controller

A combination of the fuzzy logic controller and the PID controller is presented in this work. The structure of the **FLPID** (Fuzzy Logic PID) controller consists of two parts, the classical fuzzy logic controller and PID. The block diagram of the above controller are given in [7] as shown below:

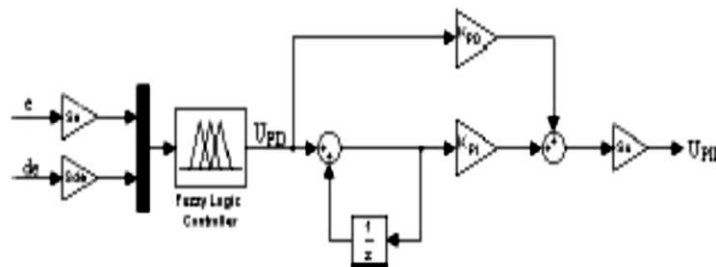


Fig. 2 Block diagram of the two input Fuzzy PID structure

## 4- Simulation and Results

In this study, simulation are conducted by using the Simulink package in Matlab environment for investigating the performance of the passive and active control of linear and nonlinear quarter car model. The simulink block diagram of a two degrees of freedom nonlinear quarter car model simulated for the purpose of the current investigation is shown in Fig.(3). While the simulink block diagram of the linear model is given in Fig.(4). In order to verify the effectiveness of the simulations, the numerical values of the system parameters are listed in Table II [7].

TABLE II  
PARAMETERS OF THE MODEL

Body mass - $m_b$	295 kg	Damping of suspension - $c_b$	1000 N.s/m
Wheel mass - $m_w$	45 kg	Nonlinear stiffness constant - $k_b^a$	$16 \times 10^3$ N/m <sup>3</sup>
Stiffness of the body - $k_b$	16 000 N/m	Nonlinear damping constant - $c_b^a$	600 N.(s/m) <sup>3</sup>
Stiffness of the tire - $k_w$	160 000 N/m	Friction coefficient - $\mu$	1

The block diagram of the linear and nonlinear passive control model is shown in Fig.(5). The simulation results of passive control of the linear and nonlinear models are shown in Fig.(6). The figure presented the behavior of each model separately. Although the behavior of the nonlinear model showed more oscillations than the linear model, this is expected because the effect of the coulomb friction, but comparing the maximum amplitudes of the two models, it is clear that the nonlinear model gives 10 times less amplitude than the linear model. The block diagrams of the linear and nonlinear active PID control model is shown in Fig.(7). The simulation results of this model are shown in Fig.(8). It is clear from this figure that the amplitude and time of the nonlinear PID model is less than that of the linear PID model, and that mean the nonlinear model is better controlled by PID than the linear model. Fig.(9) shows the block diagram of linear and nonlinear active Fuzzy control model. The simulation results of this model are presented in Fig.(10). It clear from this figure that the nonlinear model response is more controllable using this controller than the linear model. The active Fuzzy logic PID control model block diagram is illustrated in Fig.(11). The response of the models for the above control method are shown in Fig.(12). It clear from this figure that the behavior of the nonlinear model is much better than the behavior of the linear model. Finally model comparisons is carried out in this study to show the responses of the



linear and the nonlinear car suspension to the different control methods. Figures (13) and (14) illustrated these responses for linear and nonlinear models respectively. The figures shows that the fuzzy-PID control method is more efficient in controlling the car suspension system.

## **5-Conclusions**

From the above investigation, the following conclusions can be drawn:

- 1- The nonlinear model is more capable in describing the response of the quarter car suspension system for passive and active control methods.
- The fuzzy-PID control method is more stable for controlling the car suspension system than other investigated methods.
- 3- Finally the comparison between the linear and nonlinear models is the first to be carried out in this research and no such a comparison was found in the literature.

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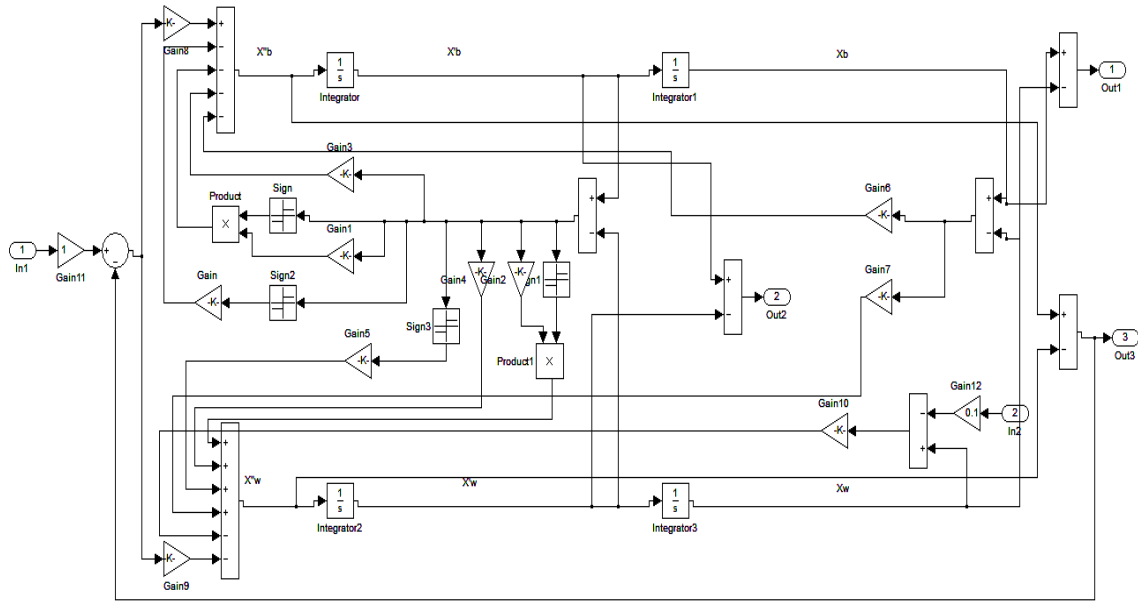


Fig.(3) Simulink block diagram of nonlinear quarter car passive model.

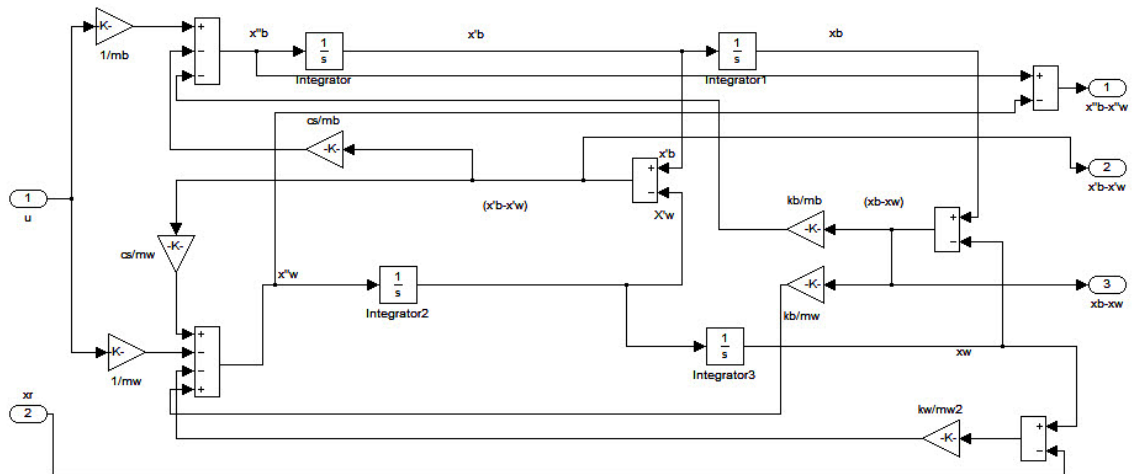


Fig.(4) Simulink block diagram of linear quarter car passive model.

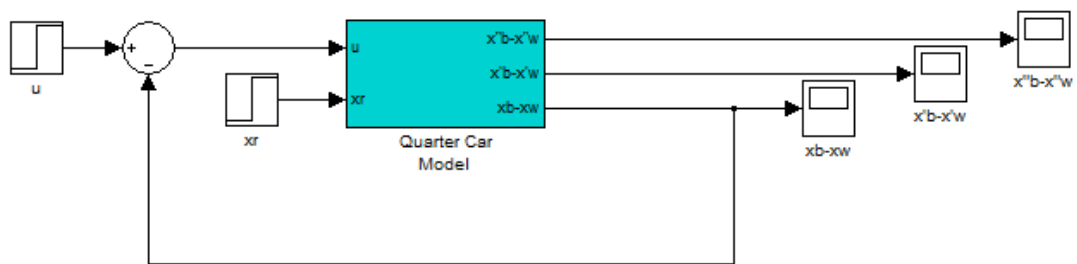
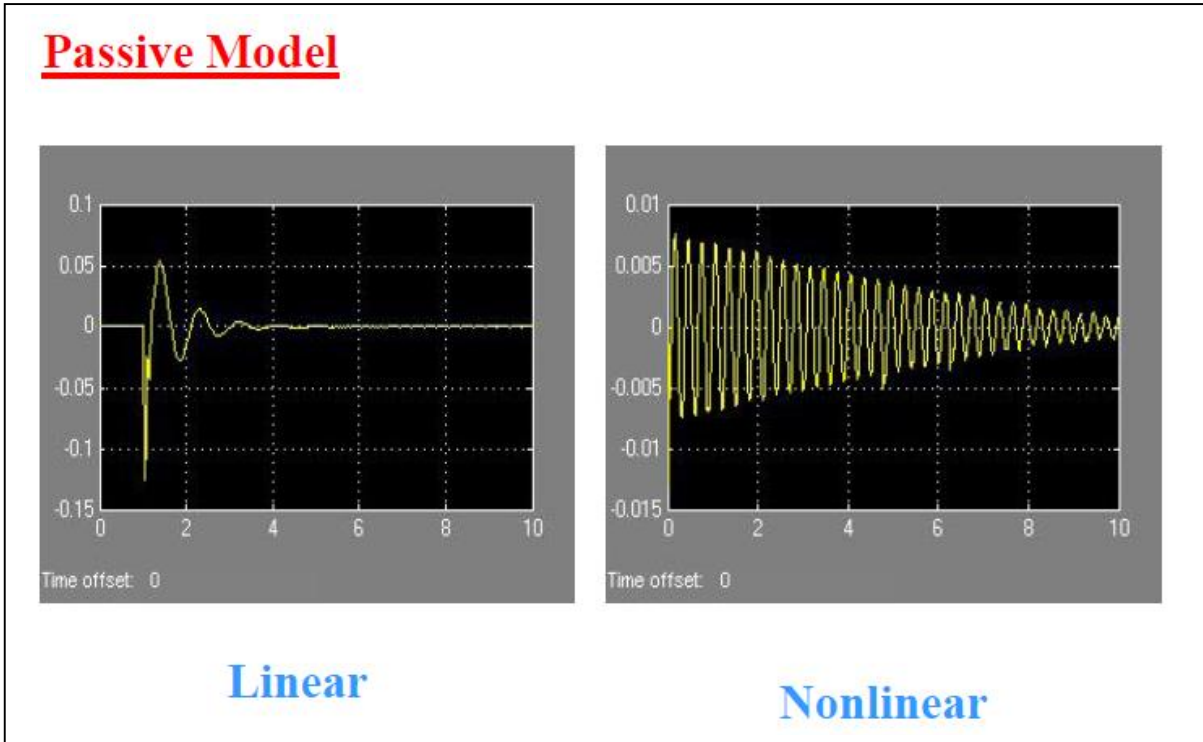
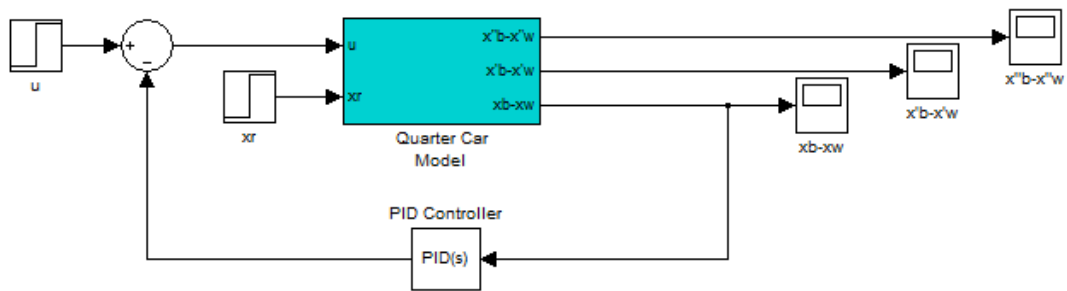


Fig.(5) Simulink Block Diagram of linear and nonlinear quarter car Passive models.

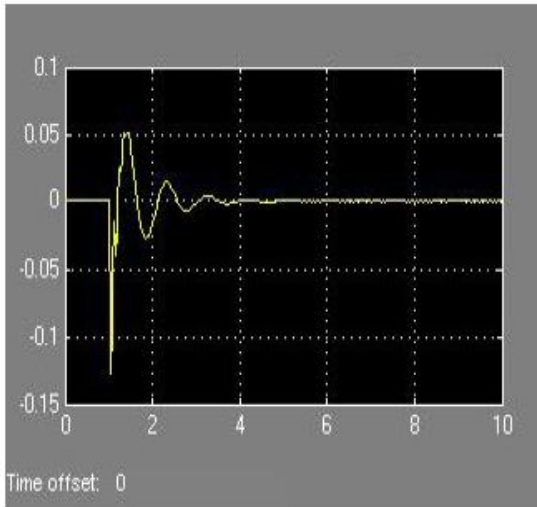


**Fig.(6) Model Response of linear and nonlinear quarter car passive models.**

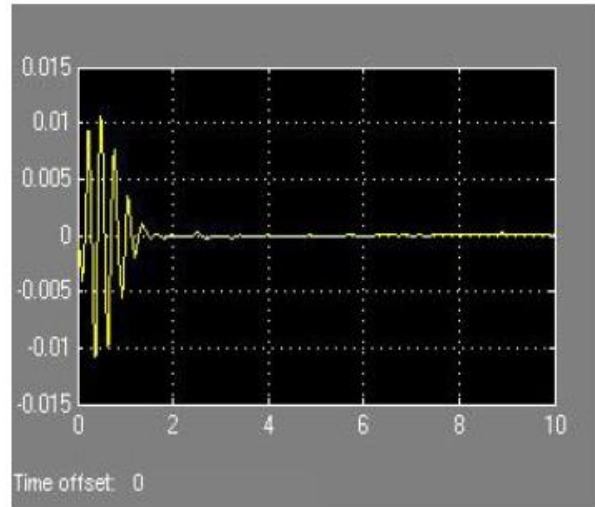


**Fig.(7) Simulink Block Diagram of linear and nonlinear quarter car active PID models.**

## PID Model

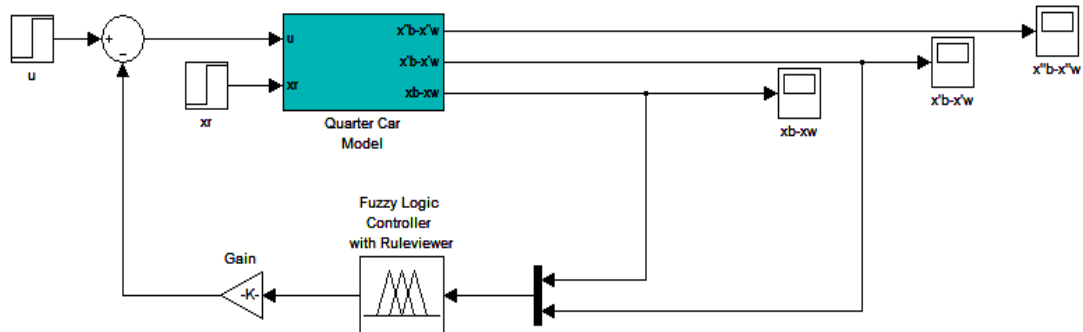


**Linear**

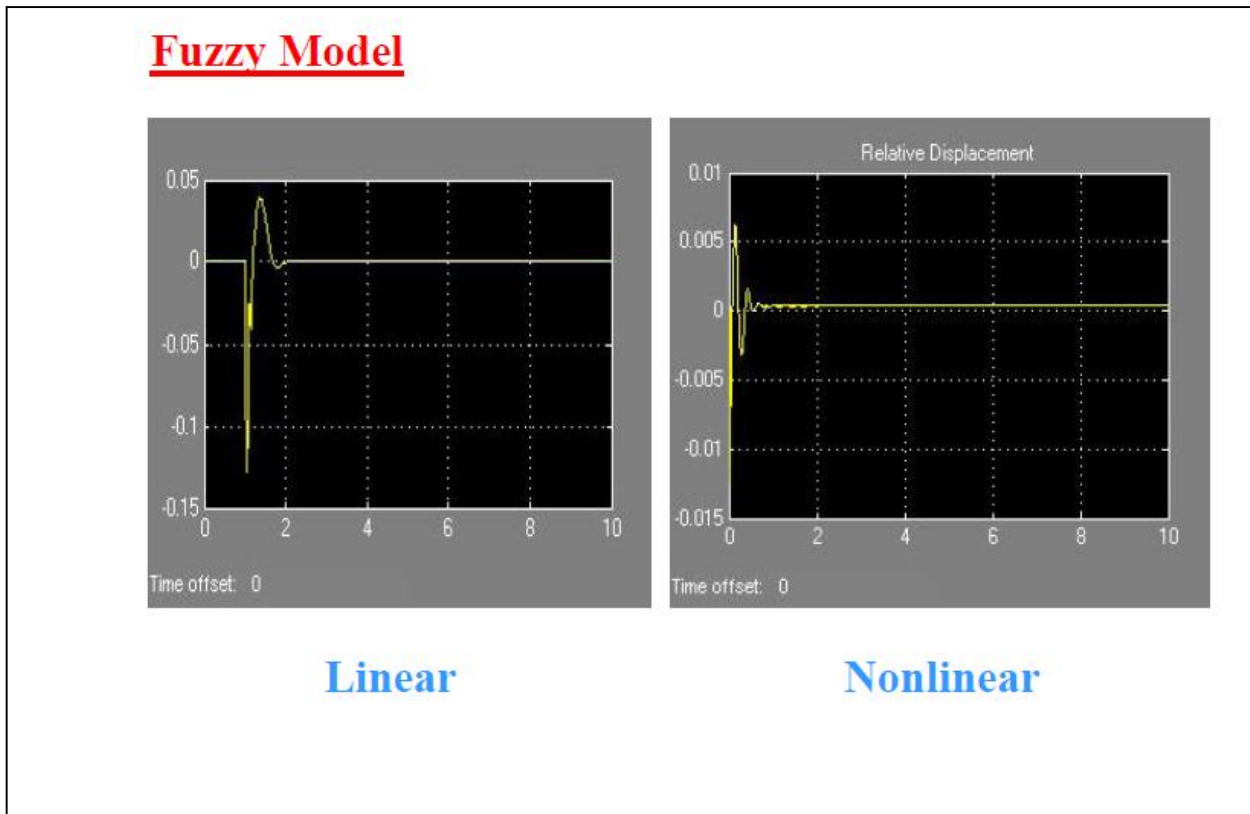


**Nonlinear**

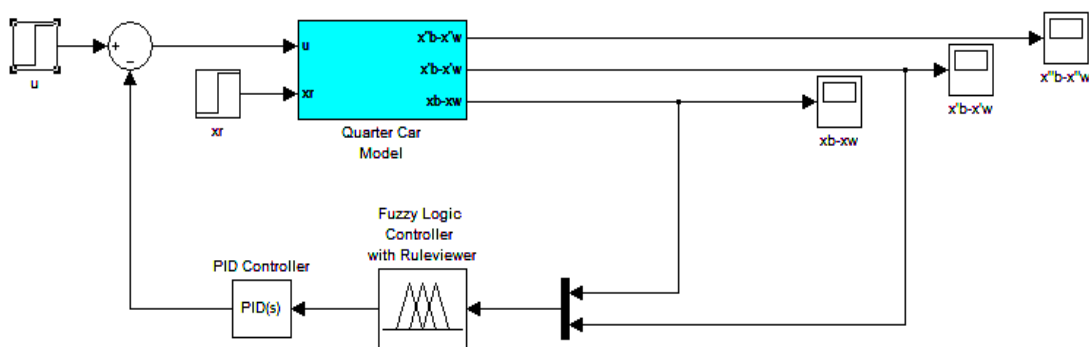
**Fig.(8) Model Response of linear and nonlinear quarter car active PID models.**



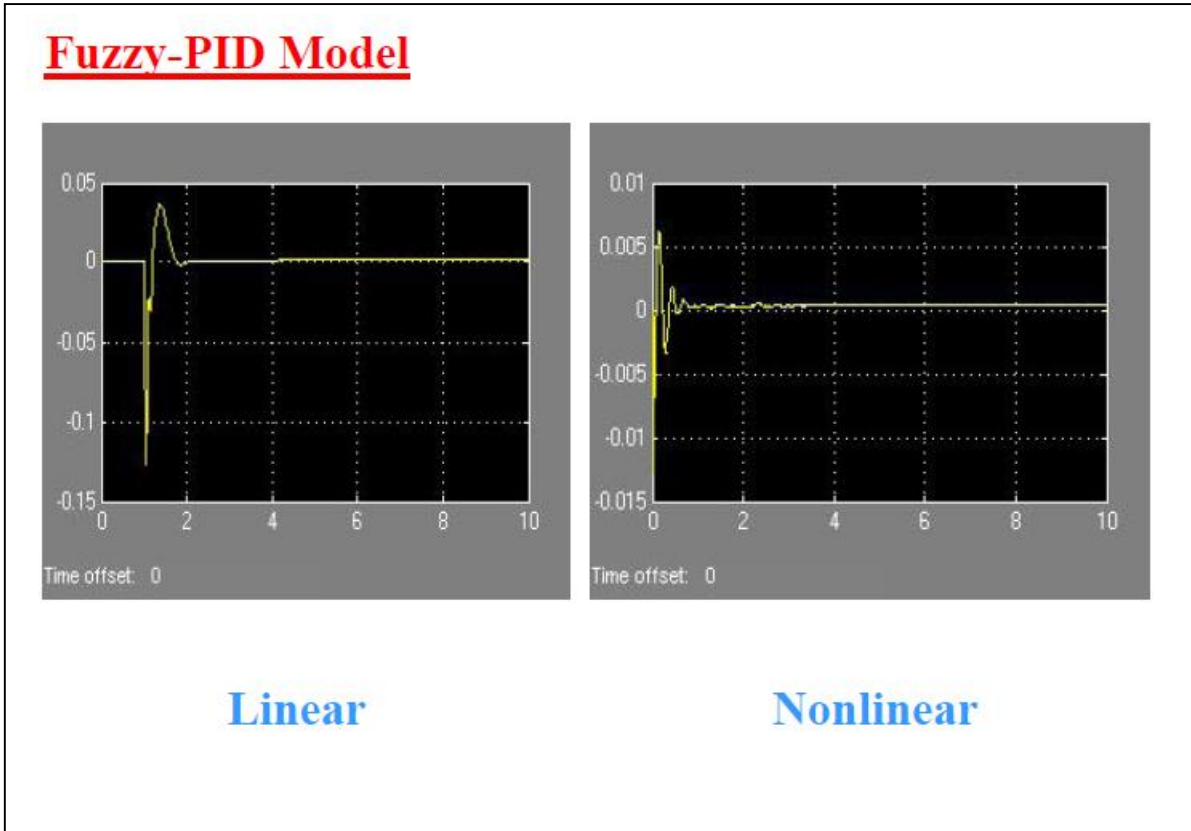
**Fig.(9) Simulink Block Diagram of linear and nonlinear quarter car active Fuzzy models.**



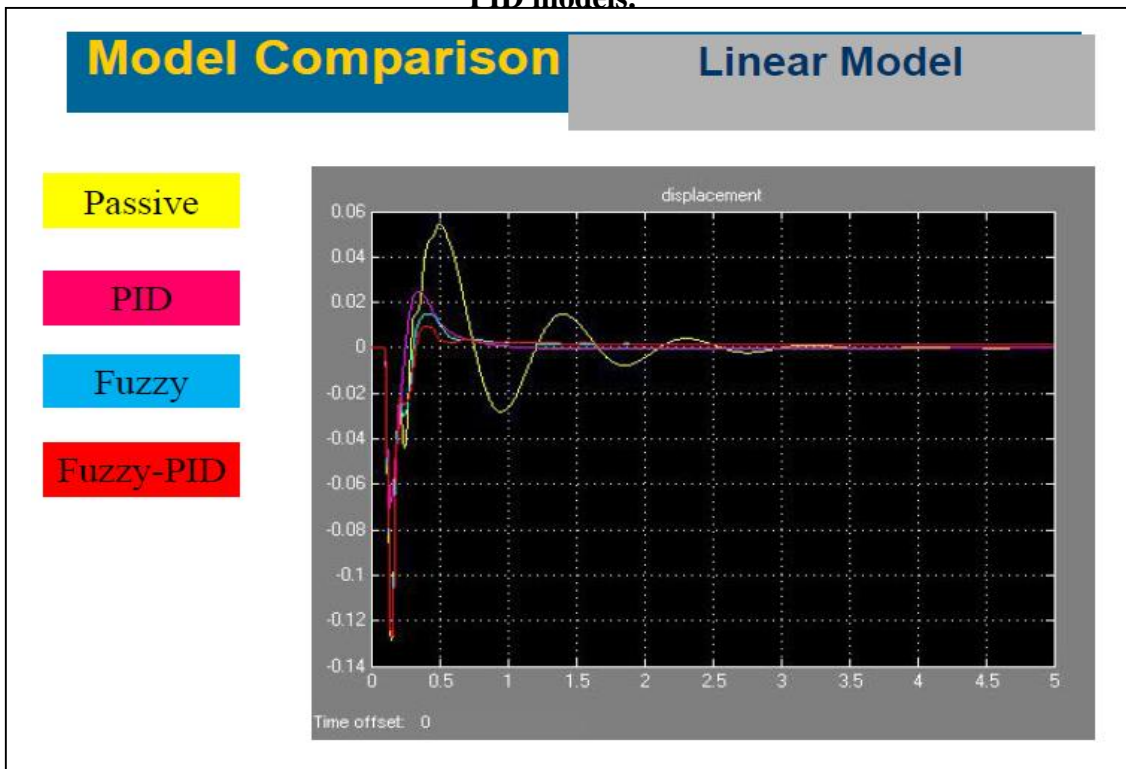
**Fig.(10) Model Response of linear and nonlinear quarter car active Fuzzy models.**



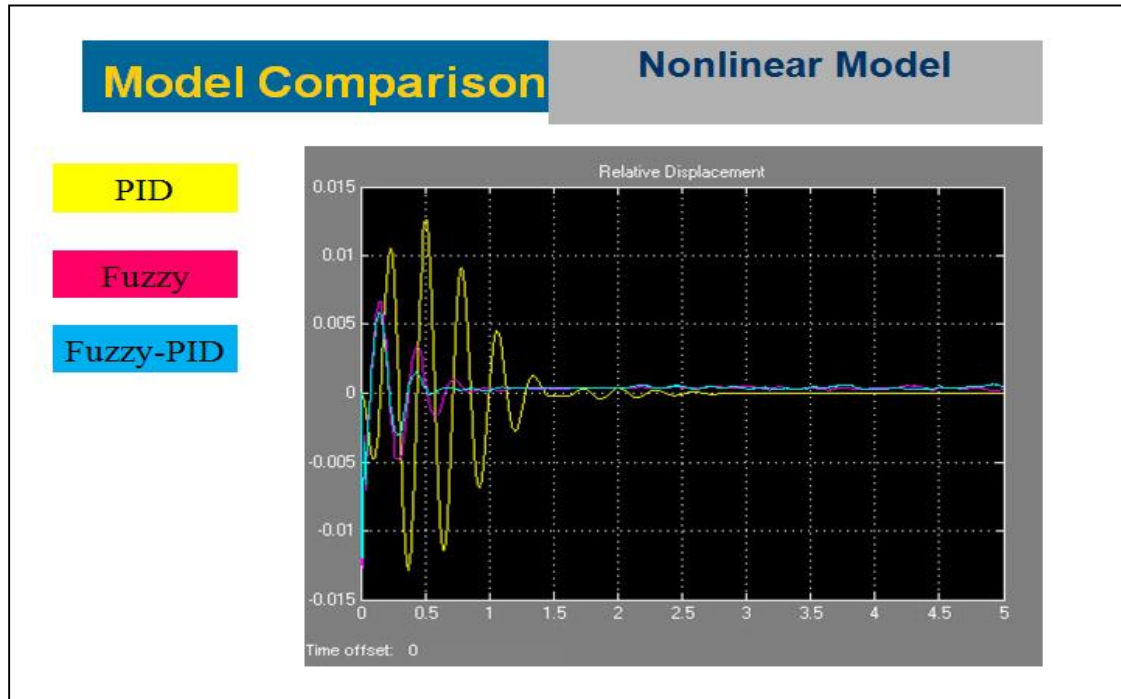
**Fig.(11) Simulink Block Diagram of linear and nonlinear quarter car active Fuzzy PID models.**



**Fig.(12) Model Response of linear and nonlinear quarter car active Fuzzy PID models.**



**Fig.(13) Model comparison of linear quarter car model with different control methods.**



**Fig.(14) Model comparison of nonlinear quarter car model with different control methods.**