Analysis of Turbulent Free Convection in Enclosure with Conductive Partitions

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Abstract: A numerical method is presented to investigate the turbulent free convection inside an enclosure with partitions. The conductive partitions were located at the bottom wall. The Navier-Stokes and energy equations besides to the kinetic and dissipation equations were discretized by using a finite volume method. The solution of these equations was made via constructing a Fortran 90 computer program. The turbulence in the flow was modeled by using a k-ε model. The obtained results show that the conductive partitions represented a key factor for enhancing the rate of heat transfer. Also the results showed that the partitions height has more effect on enhancing the rate of heat transfer compared with the partitions thickness. The height of partitions were ranged from 0.25 ≤ h ≤ 0.65 and the width from 0.07 ≤ w ≤ 0.13. The enclosure was partially heated and the range of the considered values of Rayleigh number was up to 10^{14}. The maximum rate of heat transfer is enhanced by 35% with increasing partitions height up to h = 0.65.

Keywords: turbulent natural convection, partitions, enclosure.
An experimental benchmark study on turbulent natural convection in an air-filled square cavity was made by Amapofo and Karayiannis [1]. The cavity was differentially heated and the study was performed for a Rayleigh number range up to of $1.58 \times 10^9$. The results were obtained by measuring the local velocities and temperature at multiple locations in the cavity. The local and average Nusselt numbers and the wall shear stress besides to the turbulent kinetic energy were presented. The laminar natural convection heat transfer inside a partially divided square box was investigated by Acharya and Jetli [2]. The results verified that the thermal stratification between the divider and cold wall indicated an important role. Moreover, it was found that the influence of the divider position on the overall heat transfer coefficient was small. Markatos and Pericleous [3] investigated the buoyancy-driven laminar and turbulent flow and heat transfer in a square cavity with differentially heated side walls. They used Donor-cell differencing scheme and grid refinements were used for the studied Rayleigh numbers. The obtained results were represented in tabular and graphical forms. Bilgen [4] investigated numerically the laminar and turbulent natural convection in a differentially heated enclosure. The tested Rayleigh numbers were ranged from $10^4$ to $10^{11}$. Different values of partitions location ratio, aspect ratio and height ratio were tested. The results were documented as isotherm contours and streamlines for different values of geometrical conditions. The two dimensional numerical simulation of the two vertical plates with uniform heat generation was studied by Barozzi and Corticelli [5]. A rectangular heating block with constant wall temperature was placed in the center of the enclosure. The study was done for $Gr$ ranging from $4 \times 10^4$ to $10^8$. Khalifa and Sahib [6] studied numerically the natural convection in a rectangular enclosure fitted with adiabatic partitions. The enclosure was differentially heated. They used water as working fluid to get a Rayleigh number range of $10^{11}$ to $7 \times 10^{11}$. They obtained correlations for the studied configuration and the percentage reduction in heat transfer for each case was compared to that of a single room. Sey et al. [7] performed a numerical study on transient laminar mixed convection in a two dimensional enclosure partitioned by a conducting baffle. The interaction between the external forced air stream and the buoyancy driven flow was exhibited by streamlines and isotherms. Kuyper et al.[8] investigated the laminar and turbulent natural convection in an inclined enclosure. The angle of inclination showed a significant effect on the Nusselt number. The turbulent natural convection in a partitioned enclosure was studied by Said et al. [9]. The study presented Numerical solutions for the buoyancy driven flows in an inclined two dimensional rectangular enclosure. One of the inclined walls was heated and the other was cold. The low Reynolds number k-ε model was used to model the turbulence. The flow field and average Nusselt number was investigated for different angles of inclination and Rayleigh numbers. The Nusselt number was increased as Rayleigh number increases. The natural convection heat transfer in a partially divided enclosure was studied by Yucel and Ozdem [10]. In their results, they demonstrated that the average rate of heat transfer was decreased with decreasing number and height of partitions. Zekeriya and Ozen [11] performed a numerical study for laminar natural convection in tilted rectangular enclosures with a vertically situated hot plate. The plate was very thin and isothermal on both lateral ends and act as a heat source. Fu et al. [12] studied numerically the transient laminar natural convection in an enclosure partitioned by an adiabatic baffle. It was found that that stream function strength was strictly dependent on the position of the baffle and Rayleigh number. Khalifa and Abdulla [13] studied the turbulent natural convection in a partitioned rectangular enclosure. They performed their study for Rayleigh number range up to $1.5 \times 10^8$. The correlations and the effect of the enclosure inclination angle besides to the number of partitions on the flow and thermal fields were presented and discussed. Shi et al [14] investigated numerically the laminar natural convection inside a differentially heated square cavity with a fin on the hot wall. It was found
that that the flow field is enhanced for high Rayleigh numbers regardless of baffle length and location.

So, this work aims to enhance the scientific research in this field. The natural convection is found in many engineering applications such as cooling of electronic devices, solar collectors and thermal insulation..etc. Many authors dealt with the turbulent natural convection inside enclosures.

In this work, a numerical study has been done to cover investigating the turbulent natural convection inside a partitioned square enclosure. Three conducting partitions are fixed on the bottom wall. The working fluid was air with Pr=1. As Fig. 1 shows, the vertical walls are fixed at different isothermal temperature while the top and bottom walls are insulated. The study covered a range of Rayleigh number as $10^8 \leq Ra \leq 10^{14}$, partitions height $0.25 \leq h \leq 0.65$ and partitions width $0.07 \leq w \leq 0.13$. The aim of the present study is to show how the used conductive partition can enhance the rate of heat transfer and change the flow behaviour. The aim of the present study is to show how using conductive partitions can improve the flow and then enhance the rate of heat transfer.

![Fig. 1 the present domain of study](image_url)

2. Mathematical and Numerical Analysis

The governing partial differential equations of the turbulent flow and heat transfer for the working fluid (air). The properties of the working fluid are assumed to be constant except the density in the gravity force.

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}
\]

\[
\rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + 2 \frac{\partial}{\partial x} \left( \mu_{\text{eff}} \frac{\partial u}{\partial x} \right) + \frac{\partial}{\partial y} \left( \mu_{\text{eff}} \frac{\partial u}{\partial y} \right) \tag{2}
\]

\[
\rho u \frac{\partial v}{\partial x} + \rho v \frac{\partial v}{\partial y} = -\frac{\partial p}{\partial y} + \frac{\partial}{\partial x} \left( \mu_{\text{eff}} \frac{\partial v}{\partial x} \right) + 2 \frac{\partial}{\partial y} \left( \mu_{\text{eff}} \frac{\partial v}{\partial y} \right) + \frac{\partial}{\partial x} \left( \mu_{\text{eff}} \frac{\partial u}{\partial y} \right) \tag{3}
\]

\[
\rho u \frac{\partial T}{\partial x} + \rho v \frac{\partial T}{\partial y} = \frac{\partial}{\partial x} \left( \Gamma_{\text{eff}} \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left( \Gamma_{\text{eff}} \frac{\partial T}{\partial y} \right) \tag{4}
\]

\[
\mu_{\text{eff}} = \mu + \mu_t, T_0 = (T_c + T_h)/2, \beta = 1/T_0 \tag{5}
\]

\[
\Gamma_{\text{eff}, x} = \frac{\mu}{Pr} + \frac{\mu_t}{Pr} \tag{6}
\]
The turbulence was modeled through using the k-\(\varepsilon\) model \([15]\).

\[
\rho u \frac{\partial k}{\partial x} + \rho v \frac{\partial k}{\partial y} = \frac{\partial}{\partial x} \left( \Gamma_{eff,k} \frac{\partial k}{\partial x} \right) + \frac{\partial}{\partial y} \left( \Gamma_{eff,k} \frac{\partial k}{\partial y} \right) + G - \rho \varepsilon \tag{7}
\]

\[
\rho \frac{\partial \varepsilon}{\partial x} + \rho v \frac{\partial \varepsilon}{\partial y} = \frac{\partial}{\partial x} \left( \Gamma_{eff,\varepsilon} \frac{\partial \varepsilon}{\partial x} \right) + \frac{\partial}{\partial y} \left( \Gamma_{eff,\varepsilon} \frac{\partial \varepsilon}{\partial y} \right) + C_{1\varepsilon} \frac{\varepsilon}{k} G + C_{2\varepsilon} \frac{\varepsilon^2}{k} \tag{8}
\]

\[
G = \mu \left[ 2 \left( \frac{\partial u}{\partial x} \right)^2 + 2 \left( \frac{\partial v}{\partial y} \right)^2 + \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right)^2 \right] \tag{9}
\]

\[
\mu_t = \rho c_p \frac{k^2}{\varepsilon} \tag{10}
\]

The values of the above constants in the turbulence model are \((\sigma_k ; \sigma_\varepsilon ; C_{1\varepsilon} ; C_{2\varepsilon} ; C_\mu) = (1.0, 1.3, 1.44, 1.92, 0.09)\) respectively.

The distribution of the stream function \((\psi)\) is obtained from the Poisson equation with the boundary condition \(\psi = 0\) at the solid walls.

\[
\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \tag{11}
\]

The distribution of temperature through the conducting solid baffles is gained by solving the steady state heat conduction equation using finite difference method.

\[
\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0 \tag{12}
\]

2.1. Boundary Conditions

To obtain the target results according to the physical problem, the following boundary conditions are imposed:

At the walls: \(u = v = 0\). and wall functions laws were incorporated to deal with the near wall grid points \([15]\).

For vertical walls: at \(x = 0, T = T_h\), at \(x = L, T = T_c\)

For the horizontal walls, \(\frac{\partial T}{\partial x} = 0 \quad \text{at} \ 0,y) H\).

The local dimensionless Nusselt number along the left vertical hot wall is obtained as follows:

\[
Nu = \frac{\partial \theta}{\partial X} = \frac{\partial T}{\partial x} \frac{H}{T_h - T_c} \tag{13}
\]

At the partitions boundaries:
where \( k_f \) and \( k_s \) are the thermal conductivities of the fluid and solid respectively, \( n \) is a unit normal vector.

The Nusselt number is a function of grid points and Rayleigh number. The number of grid points for \( 10^8 \leq Ra \leq 10^{10} \) is 41×41 and for \( 10^{10} \leq Ra \leq 10^{14} \) is 84×82. It is important to mention here that the increase in Ra requires more grid points and computational time to gain converged solutions. The high density of grid points for all the studied Rayleigh numbers was concentrated near the bounded walls.

The governing equations of continuity, momentum and energy for the working fluid (air) inside an inclined partitioned square enclosure were discretized to algebraic equations through using a finite volume method while that of steady state conduction equation is by a finite difference method. The gained algebraic discretized equations were solved by using a semi-implicit line by line Gause elimination scheme. Non-uniform grids in all directions were used and the densities of these grids were used near the bounding walls where steep gradients of the dependent variables are important value. The computational grids are staggered for vector variables and assigned in their original positions for the scalar variables. Because of the inherent coupling and non-linearity in the governing equations, underlaxation factors were used. The factors used for velocity components, energy and turbulence quantities are 0.5, 0.8, 0.7 respectively. A computer program is developed to get the results using the pressure velocity coupling (SIMPLE algorithm)[16]. The residual sum for each of the variables is computed and stored at the end of each iteration. The criteria \( \text{Max} |\phi^i(i, j) - \phi^{i-1}(i, j)| \leq 10^{-5} \) was the convergence criteria required for all dependent variables. The accuracy of the present results were judged through comparison with published results shown in Fig (10).

3. Results and Discussion:

The obtained results of the present work have been reported as stream function distribution, temperature distribution and Nusselt number variation. The study covered a range of Rayleigh number up to \( 10^{14} \).

Fig. 2 shows the distribution of stream function for different values of dimensionless partitions height. It is evident that the partition height has a noticeable effect on distribution of stream lines and generated vortices. As \( h=0.25 \), three vortices besides to the secondary flow are formed behind the partitions while a large vorticity is found above the partitions. The stream lines are very closer to each other in the region above the portion especially on the first one and that lead increase the velocity. However this accelerating in the flow is decelerated after the first portion due strong recirculation behind this portion. The recirculation regions behind these partitions are expected to increase the heat losses and consequently enhance the rate of heat transfer. However the disadvantage of using these partitions is distorting the cone region. When the partition height increases to \( h=0.45 \), the site and location of the resulted vortices are became larger and the stream lines are shifted away from the insulated bottom wall. The vortices behind these partitions made the stagnation cone region to be smaller and consequently affecting the temperature distribution. This trend is enhanced as \( h=0.65 \) where the stagnation core region is disappeared and number of vortices is increased. The contours of dimensionless temperature distribution are presented in Fig. 3. It is clear that the heat is transferred from the hot wall to the cold wall and partitions through the working fluid (air) and conduction heat transfer occurs through the solid partitions. The rate of heat transfer is little at the third partition and in the region behind the third partition. This expected due to
weak recirculation behind this partition which made the working fluid to be trapped in this region. This trend is found in (b) and (c). It is expected that the rate of heat transfer is increased as the height of the partition is increased. The partition distorts the isotherm lines and the convection heat transfer from the hot fluid to the solid partition is decreased after the first partition.

The effect of Rayleigh number on stream function distribution is found in Fig. 4. It can be seen that the number of vortices and strength of these vortices are increased due to increase the Rayleigh number. The number and size of these vortices are changed as Rayleigh number increases. It can mention here that the cause behind increasing the strength and size of vortices is due to increase convection currents as a result to increase Rayleigh number.

The effect of Rayleigh number on dimensionless temperature distribution is found in Fig. 5. When Rayleigh number exceeds $10^8$, the slope of isotherm lines is increased and that leads to enhance the rate of heat transfer as shown in the next section. Also this parameter affected the heat transfer through the partitions as shown in (b) and (c). Increasing Rayleigh number leads to increase buoyancy forces and consequently acceleration of the fluid motion and enhancing the rate of heat transfer. However this trend is affected by the value of height of the partitions as is explained in the next section.

The effect of Rayleigh number on the local rate of heat transfer for different values of partitions height is shown in Fig. 6. It can be seen that the trend of local Nusselt number is noticeably changed as Rayleigh number increases for the considered values of partitions height. This behaviour trend is concentrated above the partitions for $0.5 \leq Y \leq 1$. It can be seen that the rate of heat transfer is significantly increased as Rayleigh number and partitions height increase. However when $Ra>10^{10}$, this trend is reflected for at $0 \leq y \leq 0.5$. As the figure shows the values of local Nusselt number on the left hot wall is decreased sharply for $y>0$ and this behaviour is reflected sharply as Rayleigh number increases especially at (c).

Fig. 7 shows the effect of Rayleigh number on the local rate of heat transfer on the left hot wall for the considered values of partitions height. It can see that the local rate of heat transfer is increased significantly as Rayleigh number increases. The local rate of heat transfer is enhanced by nearly 15% when $Ra=10^{14}$ for $y<0.25$ and this rate is increased for $y>0.25$ and it reaches 45% at $0.75 \leq y \leq 1$. This large increase of local Nusselt number becomes very little as $Ra\geq10^{12}$. The trend of local Nusselt number variation is the same for all the partitions height.

The effect of partitions width on stream function and temperature distribution is described in Fig. 8 and Fig. 9 respectively. In Fig. 8, it can be seen that the partitions width affected the size of the resulted vortices. However there is no significant change recorded in number of these vortices for $0.03 \leq w \leq 0.07$. This trend is applicable for the other considered values of partitions height. In Fig. 9, the isotherm lines distribution shape demonstrates that the heat is transferred from the hot wall to the cold one and there is a heat transfer occurs by conduction and convection through the solid partition changes occurs. However there is no significant change occurs in the distribution of isotherm lines computed with that of partitions height.

The validation of the present code is examined with published studies [1] of this field and acceptable agreement has been obtained as shown in Fig. 10.

Fig. 10 shows the effect of partitions width on variation of local variation Nusselt number. It is shown that there is a slightly increase in Nusselt number with increasing partitions width for $Y>0.6$. It can be demonstrated here that the Nusselt number is increased up to $W=0.1$ for $Y<0.6$ after that it decreases. However it is noted that the effect of partitions width on variation of local Nusselt number is less than that of partition height.

4. Conclusions:

The following conclusions can be obtained from the present work:

1. The rate of heat transfer is enhanced as partitions height increases.
2. There is no significant change in heat transfer as the partitions thickness increases.
3. The partitions shifted the thermal boundary layer on the lower wall and this trend affected the variation of local Nusselt number.
4. The effect of partitions on enhancement of heat transfer is clearer at low values of Rayleigh number compared with those of high Rayleigh numbers. The rate of heat transfer is enhanced as Rayleigh number increases.

5. References

Nomenclature

\( \alpha \) thermal diffusivity, \( m^2/s \)
\( G \) generation term by shear, \( kg/m.s^3 \)
\( h \) relative height of the partition \((L_1/H)\)
\( H \) height of the enclosure, \( m \)
\( k \) turbulent kinetic energy, \( m^2/s^2 \)
\( L_1 \) length of the partition, \( m \)
\( Nu \) local Nusselt number
\( Nu_{av} \) average Nusselt number
\( P \) pressure, \( N/m^2 \)
\( Pr \) Prandtl number
\( Ra \) Rayleigh number \( \left( \frac{g\beta H^4(T_h - T_c)}{a v} \right) \)
\( T_C \) cold wall temperature, \( ^\circ C \)
\( T_h \) hot wall temperature, \( ^\circ C \)
\( x, y \) Cartesian coordinates, \( m \)
\( X \) dimensionless Cartesian coordinate \( \left( \frac{x}{H} \right) \)

Greek symbols

\( \epsilon \) turbulence dissipation rate, \( m^2/s^3 \)
\( \mu \) dynamic viscosity, \( N.s/m^2 \)
\( \mu_t \) turbulent viscosity, \( N.s/m^2 \)
\( \theta \) dimensionless temperature \( \left( \frac{T - T_c}{T_h - T_c} \right) \)
Fig. 2. Effect of partitions height on stream function distribution for Ra=10E8 and w=0.1
Fig. 2. Effect of partitions height on dimensionless temperature distribution for Ra=10E8 and w=0.1
Fig. 4. Effect of Ra on stream function distribution for h=0.45 and w=0.1
Fig. 5. Effect of Rayleigh number on the dimensionless temperature distribution for $h=0.45$ and $w=0.1$
Fig. 6. Variation of the local Nusselt number on vertical hot wall for different values of Rayleigh number and $w=0.1$.
Fig. 7. Effect of Rayleigh number on the local Nusselt number on vertical hot wall for different values of partitions height and $w=0.1$
Fig. 8 Effect of partitions width on stream function distribution for Ra=10E8
Fig. 9 Effect of partitions width on dimensionless temperature distribution for Ra=10E8

a. W=0.07

b. W=0.1

c. W=0.13
Fig. 10. Effect of partitions width on variation of local Nusselt number on hot wall for Ra=10E8.