University of Thi_Qar Journal for Engineering Sciences http://www.doi.org/10.31663/tqujes.11.2.398(2021) Vol 11.2(April 2021)

Study the performance of microchannel heat exchanger with combined effect of PDF and PDF and different flow arrangements

Dounia A. Mohammad[†], Mushtaq I. Hasan[‡]and Ahmed J. shkara[†]

[†], Mechanical Engineering Department, College of Engineering, University of Thi-Qar,

^{*}Mechanical Engineering Department, College of Engineering, University of Thi-Qar, Email.

[†] Mechanical Engineering Department, College of Engineering, University of Thi-Qar , Email.

Abstract

This paper numerically investigated the square microchannel heat exchanger (MCHE) at hydraulic diameter 20 μ m and the diluted water 1:1 potassium chloride (KCl) solution as a working fluid at ionic concentration 10^{-6} M with using silicon microchannel. A comparison has been made between parallel and counter flow with two methods of driven flow of liquid through MCHE (pressure driven flow PDF and electroosmotic flow EOF) with using two values of zeta potential (-75, -200 mv) to investigate the effect of zeta potential on performance of parallel and counter flow. In this paper solved a three dimensional Poisson-Boltzmann equations and Navier-stoke equations with electric field which applied on electrolyte solution have been solved using the finite volume scheme. The results showed that, the total pressure drop increased at combined EOF/PDF compared with pure pressure driven and it clear in parallel flow more than counter flow. Heat transfer rate for counter flow is higher in parallel flow. That the increasing in zeta potential of surface charge caused an increase effect of EDL due to increase in Helmholtz-Smolochowski velocity.

Keywords: microchannel heat exchanger (MCHE), zeta potential, electroosmotic flow, pressure driven flow, numerical investigation, Poisson Boltzmann equation.

1. Introduction

Microchannel heat exchanger is one of the important applications of microfluidic devices. The growing demand for knowledge in the heat transfer and flow through microfluidic devices due to the rapid evolution of (MEMS) and important knowledge in the liquid flow in structure and design of microchannels [1], there are many researcher in literature studied the microchannel with EDL effect such as:

(2000) [2] experimentally and Liqing Ren et al. theoretically studied for impact of electro-viscous by EDL effect on single microchannels. They used two ionic concentration 10^{-2} , 10^{-4} of the diluted 1:1 potassium chloride (KCl) solution and three sizes of channel 14.1, 28.2, 40.5 µm. They conclude that, the relationship between pressure gradient $\frac{dP}{dx}$ and Reynold number strongly dependent on the hydraulic diameter and ionic concentration. The increasing in streaming potential lead to increase in electroviscous, the diluted liquid that has highest ionic concentration (10^{-2}) M and zeta potential has streaming potential less. They found a agreement between theoretical and experimental results. Amanifar et al. (2007) [3] numerically studied of EDL impact at pressure driven flow for rectangular microchannel heat sink using Navier-stokes equations (2D) with EDL effect and Poisson-Boltzmann equation (2D). The walls of microchannel is silicon and dilute liquid as cooling liquid. They found that, the impact of electric double layer cannot be negligible at ionize liquid because lead to reduce in flow velocity of liquid and increase in pressure difference and hence effect on efficiency, the efficiency at without EDL higher than it with EDL effect. Shokouhmand and Jafari (2010) [4] numerically investigated of mixed convection with electric double layer effect of vertical microchannel as shown in fig. (1) with symmetric wall temperature. They solved 2D Poisson Boltzmann equation and also Nernst Plank equation to determine ionic concentration field. Navier stokes equations with effect of electrical field solved by finite volume method. Their results indicate to, at increasing the Reynold number with constant value of electric potential this caused in tends of flow to poiselluie flow (parabolic shape) with increased in Nusselt number. The Grashof number has important effect at increment, such lead to effect on heat transfer and fluid flow.



Fig.1 shows the vertical microchannel

Davong Y. (2011) [5] analytical solution of velocity distribution and potential for mixed pressure gradient /electroosmotic flow in microchannel. The mathematical model to simulate fluid flow which consist the Navier stokes equations and Poisson Boltzmann equation which solved by using finite element method in Matlab software. Their findings indicate to, at combined EOF/PDF the distribution of velocity is parabolic and compound of plug-like when the fluid is steady state, either in case pure EOF the distribution of velocity is plug-like and similar to the EDL potential profile. Their results supply the guidelines to applications of combined flow (EOF/PDF) in microchannel chips. Bera S. and Bhattacharyva S. (2013) numerical studied of combined pressure [21] gradient/electrosmotic driven flow through single micro/Nano channels by solved Navier stokes equations with Poisson Boltzmann equation and Nernst equation with comparison the results between two methods at thin EDL and thick EDL. The user liquid is incompressible Newtonian (Nacl with water). They concluded that, using large value of pressure gradient lead to negligible the electrokinetic effect and is a similar to Poiseulle flow, combined EOF/PDF effect mainly depended on the EDL thickness.

Nader N. (2016) [7] analytical studied for conjugated conduction-convection heat transfer of parallel plate with applied heat flux on external surface of channel with EDL influence to take it into account. Energy equation investigated in two cases, the first case is solved to find the temperature distribution and second case solved by infinite series. They found that, reduced in the heat transfer rate specially at higher zeta potential, at reduced thermal conductivity ratio in thermally developed boundary lead to decreasing in heat transfer rate and the Nu decrease at higher zeta potential. Ali et al. (2019) [8] numerically investigated of surface roughness effects of parallel flow in square microchannel heat exchanger on thermal and hydrodynamic performance at various hydraulic diameters, the working fluid is water. They conclude that , the surface roughness impact lead to improve the thermal performance and increasing in pressure difference and hence the effect on pumping power. The effect of roughness reduced with increment Re and hydraulic diameter. Yawen X. et. al. (2020) [9] numerical model two dimensional of electroosmotic flow in micro-mixer as shown in fig. (2-3) by using COMSOL multi physics software to simulate Navier stokes equations. The fluid is water with Nacl solution with the following assumptions: dynamic viscosity and density of electrolyte solution are constant, incompressible Newtonian liquid, fully developed steady state no wall slip and the impacts of gravity field and temperature in EOF are unconsidered. They concluded that, increased in mixing efficiency at micro mixer with electroosmotic by alternating electric field due to generate varying electric current in micro mixer, at increased the applied field potential caused in increased in mixing efficiency and then decreased therefore must be select suitable electric field to improve the mixing efficiency of micro mixer with EOF.

In this study a comparison will be made between two driven flow methods pure pressure driven and electroosmotic flow/pressure driven flow at two types of flow parallel and counter flow with two values of zeta potential.

2. Problem description

In this paper the effect of electric double layer has been studied on the performance of microchannel heat exchanger at two types of flow parallel and counter flow for (MCHE) shown in Fig.1, for simplification purposes one unit of microchannel heat exchanger consists of hot and cold channel shows in fig. 2 is studied, the length of heat exchanger is 1mm and its hydraulic diameter (20µm), the thickness of the wall between the hot channel and cold channel is 3µm, Reynold number selected with range (50-150) and the temperature of inlet hot and cold are 373 K, 293 K receptivity. The electric potential effect of the surface of the channel (-75 mvolt, -200 mvolt), ionic concentration of liquid is 10^{-4} , field electric Ez applied on working fluid is $10^{6} \frac{V}{m}$.



Fig.2A schematic model of microchannel heat exchanger.



Fig.2B a unit of heat exchanger consists of hot and cold channels at parallel flow [10].

2.1. Electric double layer

The zeta potential is surface charge of solid wall of microchannel, the liquid has a few quantities of ions, this lead to attraction the counterions in dilute liquid to electrostatic charges of surface to create electric field, the order of electrostatic charge and ions of liquid is called (EDL). There are two types of ions in term of the motion, for compact layer (the layer near of surface) the ions are immobile and in diffuse double layer (DDL) the electric field less influence to electric field on the ions (mobile). When the dilute liquid flow during the microchannel, the mobile ions of the electric double layer create an electric current (streaming current) to flow with liquid flow. Gathering of the ions in direction of flow sets up an electric potential and electric field together known streaming potential, the streaming potential neglected in case electroosmotic flow as a result of applied electric flied more higher than generated streaming potential. The maximum value of thickness of the electric double layer (EDL) 1µm depending on the properties mentioned above. (a) ionic concentration, (b) temperature of dilute liquid and (c) zeta potential of surface. To shows characterize of the electric double layer (EDL) effects used the Debye-Huckel parameter k [11]. Fig. 3 shows the formation of EDL on wall of channel. 1/k referred to as thickness of the electric double layer (EDL).



Fig.3 Scheme formed of electric double layer on surface wall [12].

3. Mathematical formulation:

For 3D steady state, incompressible and laminar flow, the following equations are solved to calculate the distributions of velocity, temperature and EDL for parallel and counter flow through microchannel heat exchanger.

Poisson's equation :

According to the theory of electrostatics, the relationship between Ψ and ρe is given by the Poisson's equation [13], which for a rectangular channel

$$\frac{\partial^2 \Psi}{\partial x^2} + \frac{\partial^2 \Psi}{\partial y^2} + \frac{\partial^2 \Psi}{\partial z^2} = \frac{\rho e}{\varepsilon \varepsilon_0}$$
(1)

- $\epsilon \ \ \, is the dielectric constant of the medium$
- ϵ_o is the electric permittivity of a vacuum.

 ρe is the net volume charge density, we have

$$ni = n_o iexp(-\frac{zie\Psi}{kbT}) \tag{2}$$

$$\rho e = (n^+ - n^-) = -2n_o zesinh(\frac{ze\Psi}{kbT}) \quad (3)$$

 n^+ and n^- are concentration of cations and anions ,respectively

- Kb Boltzmann's constant =1.3805 * 10^{-23} /mol⁻¹K⁻¹
- *e* electron charge = $1.6021 * 10^{-19}C$

noi bulk concentration

Poisson-Boltzmann eq. become

$$\frac{\partial^2 \Psi}{\partial x^2} + \frac{\partial^2 \Psi}{\partial y^2} + \frac{\partial^2 \Psi}{\partial z^2} = \frac{2n_o ze}{\epsilon \varepsilon_o} \sinh \frac{ze\Psi}{KbT}$$

Continuity equation: [15]

$$\frac{\partial \mathbf{u}}{\partial \mathbf{x}} + \frac{\partial \mathbf{v}}{\partial \mathbf{y}} + \frac{\partial \mathbf{w}}{\partial \mathbf{z}} = \mathbf{0}$$
(4)

Momentum equations :

x- direction

$$u\frac{\partial u}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial w}{\partial z} = \frac{\mu}{\rho} \left(\frac{\partial^{2u}}{\partial x^{2}} + \frac{\partial^{2u}}{\partial y^{2}} + \frac{\partial^{2u}}{\partial z^{2}} \right) - \frac{\mu}{\rho} \frac{\partial p}{\partial x}$$
(5)

y- direction

$$u\frac{\partial v}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial v}{\partial z} = \frac{\mu}{\rho} \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} \right) - \frac{\mu}{\rho} \frac{\partial p}{\partial y}$$
(6)

A body force in Z- direction originates due to the presence the electric field ($Ez \rho_e$) Dounia A. Mohammad [†], Mushtaq I. Hasan [‡]and Ahmed J. shkara[†]

$$u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} = \frac{\mu}{\rho} \left(\frac{\partial^{2w}}{\partial x^{2}} + \frac{\partial^{2w}}{\partial y^{2}} + \frac{\partial^{2w}}{\partial z^{2}}\right) - \frac{1}{\rho}\frac{\partial p}{\partial z} - Ez\rho_{e}$$
(7)

Energy equation:

$$u\frac{\partial T}{\partial u} + v\frac{\partial T}{\partial v} + w\frac{\partial T}{\partial w} = \frac{k}{\rho C p} \left(\frac{\partial^{2T}}{\partial x^{2}} + \frac{\partial^{2T}}{\partial y^{2}} + \frac{\partial^{2T}}{\partial z^{2}}\right)$$
(8)

The heat transfer rate of microchannel heat exchanger between hot and cold:

$$Q = mCp\Delta T \tag{9}$$

The effectiveness(
$$\varepsilon$$
) = $\frac{Qact}{Qmax} = \frac{Ch(Thi-Tho)}{Cmin(Thi-Tic)} = \frac{Cc(Tco-Tci)}{Cmin(Thi-Tci)}$ (10)

To calculate the overall performance of MCHE, the performance index which relates the thermal and hydrodynamic performances

$$\eta = \frac{\varepsilon}{\Delta P t} \tag{11}$$

the pressure drop can be calculated from

$$\Delta Pt = \Delta Pc + \Delta Ph = (Pin, c-Po, c) + (Pin, h-Po, h)$$
(12)

The inlet velocity determine from $\operatorname{Re}\nu = \frac{\operatorname{Re}\mu}{\rho \, Dh}$ (13)

The boundary conditions

At inlet of the channel Z = 0, then U = Uin V = 0W = 0 &T = Tin.

At outlet of the channel Z = Lch and the flow is fully developed, then

 $\frac{\partial u}{\partial z} = 0$ $\frac{\partial v}{\partial Z} = 0$ $\frac{\partial w}{\partial z} = 0 \& \frac{\partial T}{\partial z} = 0$

For three cold walls adiabatic (sides walls and upper wall of cold channel) condition is applied:

$$u = 0, v = 0, w = 0, \qquad \frac{\partial T}{\partial z} = 0$$

For three hot walls adiabatic (sides walls and lower wall of hot channel) condition is applied:

$$u = 0, v = 0, w = 0$$

At x=0, $\Psi = \zeta_{o}$
x=w, $\Psi = \zeta_{o}$
At y=0, $\Psi = \zeta_{o}$

At y=h, $\Psi = \zeta_o$

4. Properties of material:

Water is a working fluid with aqueous KCL solution at the ionic concentration $(10^{-6}M)$, vacuum permittivity $8.85 \ast 10^{-12} (F/m)$, dielectric constant ϵ is 80 and $6.39 \ast 10^{-10}$ Permittivity of water (C/V*m). The properties of material for liquid and solid are listed in table (1).

Table 1 properties of used materials

material	ρ	Ср	Κ	μ
	Kg/m^3	(J/Kg.K)	(W/m.K)	Kg/m.s
Dilute	997	4182	0.605	0.9e-03
liquid				
Silicon	2329	700	148	-

5. Numerical solution:

The system of governing equations and boundary conditions are numerically solved using finite volume method (FVM). SIMPLE algorithm is used to solve the problem of velocity-pressure coupling and UDF to solve Poisson-Boltzmann equation to study effect electric double layer (EDL). The grid has been generated and the effect of mesh size has been studied, table (2) shows independent of grid size for hydraulic diameter ($20\mu m$) and its effect on the solution results and the mesh has been used for all calculation become independent of mesh as can be seen from this table. Size element equal 1 μm used in this work.

Table 2 mesh independent

Mesh size	Outlet temperature (K) of hot channel	Outlet temperature (K) of cold channel
Mesh 1 (size	318.253	347.748
element =5µm)		
Mesh 2 (size	318.287	347.573
element =3µm)		
Mesh 3 (size	318.204	347.302

Dounia A. Mohammad † , Mushtaq I. Hasan ‡ and Ahmed J. shkara †

element =1µµm)		
Mesh 4 (size	318.201	347.30
element = $0.8 \mu m$)		

6. Results and discussion:

Numerical solution of microchannel heat exchanger (MCHE) to compared between parallel and counter flow configuration, the length of microchannel heat exchanger is (1mm) and thickness of the separation wall is (3μ m). The inlet temperatures of hot and cold water used as a boundary condition are Th,in = 373 and Tc,in = 293K.

To check the numerical model validity, verification was made by solving model presented in [14] an compared the results. The model presented in [14] is a square microchannel with hydraulic diameter 25 μ m, length 1000 μ m $\Delta P/dx = 10^5 \frac{Pa}{m}$, inlet temperature of 289 K. The concentration 10^{-4} , 10^{-5} M, zeta potential $\zeta = 150$, 200 m volt, silicon is the metal used in microchannel. **Fig. 4** shows the comparison between results of the data of [14] for non-dimensional volumetric flow rate of the microchannel with non-dimensional pressure difference and that for present numerical model. From this fig. it seen that there is a good agreement between results of present model and that for [14] and the error is equal 3.5 %.



Fig. 4 For present model and that for [14]

Fig. 5 expressed the relation between total pressure drop with Reynold number at two configurations of flow (parallel and counter flow) with two values of zeta potential (-75, -200 mv) at combined electroosmotic /pressure driven flow to investigate the EDL eefect and comparison with pure pressure driven flow (no effect of EDL). From figure noted that the pressure drop is increased as increasing the Reynold number for all types of flow due to high velocity of liquid. It can be concluded

from fig. 5 that the pressure drop for parallel flow is higher than pressure drop in counter flow, increasing in pressure different with effect of EDL (mixed EOF/PDF) and very obvious in case parallel flow due to increase in flow through hot and cold channels as a result of applied external electric field with direction of pressure driven flow, but the increment in pressure drop is very slight in counter at combined driven flow compared with pure pressure driven due to the electric field in hot channel is with direction of flow therefore caused in increased in pressure drop of hot channel, and applied electric field in cold channel is opposite to direction of pressure driven flow therefore caused a decrease in pressure drop is equal to the increasing in pressure drop in hot channel.



Fig. 5 Variation of total pressure drop (Kpa) with Reynold number.

Fig. 6 explains the variation of temperature difference (ΔT) with Reynold number for square microchannel heat exchanger with two types of flow parallel and counter flow for pure pressure driven and mixed EOF/PDF with two values of zeta potential. Can be notice from figure the following: the temperature difference reduced with increasing in Reynold number due to increasing in velocity of flow and hence effect on ΔT , the temperature difference in counter flow is higher than parallel flow due to opposite flow allow more exchange in heat between channels, also there is increased in temperature different at mixed EOF/PDF specially with high zeta potential because increase effect of EDL, the different between two types of combined or pure pressure driven become slightly with increasing in velocity of pressure driven with constant velocity of electroosmotic flow.



Fig. 6 Variation of temperature difference (K) with Reynold number.

Fig. 7 indicates the variation of effectiveness (ϵ) for microchannel heat exchanger with Reynold number at parallel and counter flow with two values of zeta potential (-75, -200mv). Notice that, the effectiveness lowered with increasing in Reynold number and the effectiveness in counter flow is higher than parallel flow due to higher the temperature difference. At EDL impact (combined EOF/PDF), the effectiveness in parallel and counter flow rise with increasing in zeta potential as a result of the increasing in zeta potential caused by increasing in velocity of elecroosmotic (Helmholtz-Smolochowski velocity) and hence increased in ratio between velocity of pressure driven to velocity of electroosmotic. The figure shows in counter flow until Re=120 there is increasing in effectiveness at mixed driven flow compared with pressure driven but, in parallel flow at Re=120 the effectiveness in case pure pressure driven is equal to almost the effectiveness for combined EOF/PDF.



Fig. 7 Variation of effectiveness with Reynold number.

The relation between performance index (η) with Reynold number at parallel and counter shown in **fig. 8.** From this figure it can be concluded that, the performance index reduced with increasing Reynold number due to increase in total pressure drop as increased the flow velocity, and the performance index in counter flow is higher from parallel flow, this because the total pressure drop in counter flow lower than parallel flow. For EDL effect, the performance index decreased with increasing in zeta potential as result of higher pressure drop with combined driven mostly at high zeta potential (-200) of silicon surface.



Fig. 8 Variation of performance index with Reynold number.

In **fig. 9** explains the variation of the heat transfer rate (O) for unit microchannel heat exchanger parallel and counter configurations with Reynold number (50-150) at two values of zeta potential. From this figure notice that, the heat transfer increased with increasing Reynold number and the heat transfer rate in counter flow is higher than parallel flow due to the heat transfer happen in a vertical direction this caused in acceleration heat transfer from hot channel to cold channel this lead to temperature difference in counter flow higher than parallel flow. For combined PDF/EOF notice that, enhancement in heat transfer rate mostly as increasing the zeta potential because the EDL field which caused in reduced in velocity with near from wall of microchannel, this means increased in exchange heat between the channels with noted this process depended on EDL thickness and surface charge.



Fig. 9 Variation of heat transfer (watt) with Reynold number.

To investigate the effect of zeta potential on combined EOF/PDF **fig. 10** shows a comparison between parallel and counter flow for the pumping power (P.P) of microchannel heat exchanger with Re at two value of zeta potential (-75, -200) at pure pressure driven and combined EOF/PDF. From figure can be conclude that, increased in pumping power as increasing the Reynold number due to increase in total pressure drop and flow velocity of liquid, the pumping power in counter flow lower than parallel flow due to the total pressure losses in parallel higher than pressure losses in counter flow. At comparison between two driven flow, from fig 10 noted that, higher in pumping power with combined EOF/PDF especially when zeta potential equal -200 mv due to increase in Helmholtz-Smolochowski velocity (EOF velocity).



Fig. 10 Variation of pumping power (watt) with Reynold number.

The variation of performance factor (ratio between Q to P.P) for microchannel with Reynold number at parallel and counter flow with two value of zeta potential shows in **fig. 11.** From figure can be seen that, the overall performance decreased with increasing Reynold number and the overall performance in counter flow higher than parallel flow due to has higher heat transfer rate and lower pumping power, the performance factor for combined driven flow higher than pure pressure driven due to increase in pumping power mostly at high zeta potential because increase in zeta lead to increase impact of EDL.



Fig. 11 Variation of overall performance with Reynold number.

Conclusions

. From the obtained results the following conclusions can be made:

1. The total pressure drop increased at combined EOF/PDF compared with pure pressure driven and it clear in parallel flow more than counter flow due to different behavior of flow in each channels.

2. Heat transfer rate for counter flow is higher of parallel flow due to the heat transfer happen in a vertical direction this caused in acceleration heat transfer from hot channel to cold channel this lead to temperature difference in counter flow higher than parallel flow.

3. The increasing in zeta potential of surface charge caused an increase in effect of EDL due to increase in Helmholtz-Smolochowski velocity.

4. The performance factor for counter flow is higher than that for parallel flow due to it has lower pumping power and higher heat transfer rate

References

[1] Qu Weilin, Gh. Mohiuddin Mala and Li Dongqing, "Pressure-driven water flows in trapezoidal silicon microchannels", Int. J. of heat transfer and mass transfer, 43, pp. 353-364, 2000.

[2] Liqing Ren, Qu Weilin and Li Dongqing, "Interfacial electrkinetic effects on liquid flow in microchannels ", Int. J. of heat transfer and mass transfer, 44, pp. 3125-3134, 2001.

[3] N. Amanifard, M. Borji and A. K. Haghi, "Heat transfer in porous media", Brazilian Journal of Chemical Engineering, Vol. 24, No. 02, pp. 223 - 232, April - June, 2007.

[4] Shokouhmand and Jafari, "Investigation of mixed convection in a vertical microchannel affected by EDL", Proceedings of the World Congress on Engineering Vol. II, London, U.K., 2010.

[5] Dayong Yang, "Analytical solution of mixed electroosmotic and pressure-driven flow in rectangular microchannels", Key Engineering Materials Vol. 483, pp 679-683, 2011.

[6] S. Bera and S. Bhatttacharyya, "On mixed electroosmotic –pressure driven flow and mass transport in microchannels", Int. J. of Engineering Science, 62, pp. 165-176, (2013).

[7] Nader Nekoubin, "A single domain formulation on conjugate heat transfer in parallel plate microchannel with electrical double layer", International Journal of thermal sciences, 110, pp206-221, 2016.

[8] Ahmed A. Ali, Mushtaq I. Hasan and Ghassan Adnan, "Study surface roughness on overall performance of parallel flow microchannel heat exchanger", University of Thi-Qar Journal for Engineering Sciences, Vol. (110), No. (1), (2019).

[9] Yawen Xue, Yi Zhou and Yemao Yin, "Study on Flow Characteristics of Microfluid in Electroosmotic Flow Mixer", Journal of Physics: Conference Series: 1549 (2020).

[10] Mushtaq I. Hasan, "Effect of variable fluid properties on the hydrodynamic and thermal characteristics of parallel flow microchannel heat exchanger", Journal of University of Thi-Qar, Vol. (10), No. (4), (2015).

[11] S. Kumar, "The EDL Effect in Microchannel Flow: A Critical Review", International Journal of Advanced Computer Research, vol.(3), No.(4), (2013).

[12] Meisam Habibi Matin and Waqar Ahmed Khan, "Entropy generation analysis of heat and mass transfer in mixed electrokinetically and pressure driven flow through a slit microchannel", Energy, 56, pp. 207-217, 2013.

[13] Mehdi Sheikhizad and Mohammad Kalteh, "Heat transfer investigation of combined electroosmotic/pressure driven nanofluid flow in a microchannel: Effect of heterogeneous surface potential and slip boundary condition", Journal Pre-proof, (2019).

[14] Reza Monazami and Mehrdad T. Manzari, "Analysis of combined pressure-driven electroosmotic flow through square microchannels", Microfluid Nanofluid, 3, pp. 123–126, (2007).

[15] E. Y. K. Ng and S. T. Tan, "Numerical analysis of EDL effect on heat transfer characteristic of 3-D developing flow in a microchannel", Numerical heat transfer, 49, pp. 991-1007, 2006.