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Mathematical Model of Single Overhead power Line in Symmetrical operation Mode of Higher Harmonic Switching

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Abstract

This paper presents a mathematical model for detecting a single-phase to ground fault by using the frequency characteristics. The variety of types of single-phase-to-ground faults on overhead power lines significantly complicates the development of one universal method for its recognition and determination of the alleged place of damage. An additional aggravating factor is the complex configuration of district electrical networks, where up to several dozen consumers can be on one feeder, and the total length of its overhead line reaches several tens of kilometers. The most dangerous demonstration of single-phase to-ground fault is the resulting overvoltage's in the network. The intermittent arc at the site of the single-phase-to-ground faults serves as a source of higher harmonics, which, in turn, due to "resonant" phenomena, can reach unacceptable values. It follows that for the correct reproduction on mathematical models of emerging phenomena, it is necessary to use the mathematical theory of long lines. Higher harmonics caused by single-phase-to-ground faults can serve not only as an indicator of the occurrence of damage, which is already being implemented in existing microprocessor relay protection terminals, but also provide information on the approximate location of the site with single-phase-to-ground fault. The mathematical model is initiated from a simplified equation for long distance transmission. To asses the accuracy of the calculation performed for the power transmission in equation its model was made in the Simulink Package and the overhead line was introduced into the model as an object with distributed parameters.

Increasing the level of short-circuit currents on the buses of the power center leads to the upsetting the resonant peak of the frequency response signal relative to the beginning of the line, which makes it preferable to use frequency characteristics relative to the end of the transmission. In this case, combinations of network parameters and the magnitude of the contact resistance are possible, when the "resonant" amplification of higher harmonics is not evident.

Keywords: Mathematical Model, single line-to-ground fault, Transmission lines, Frequency characteristics

1. Introduction

Frequency characteristic of electrical machines are actively used both to analyze the modes of their operation and to identify emerging defects **[1]**. For overhead power lines, their use is also promising, since the source of higher harmonics is created naturally- by an intermittent arc in the presence of damage. Therefore, studies are needed regarding the type of frequency characteristics (FC) in different transmission conditions **[2,3]**. Before proceeding of FC of power transmission, we will determine what factors should be taken into account in relation to the power systems: synchronous and asynchronous machines, transformers and autotransformers. First, it is necessary to reckon with the influence of the power supply network and the load. For electrical machines the frequency characteristics FC used to take in condition of disconnection from the network. For the power transmission, this is unacceptable, because when single-phase to earth fault occurs, when higher harmonics enter the line, both consumers at the step-down substations and the supply network of higher voltage remain connected **[4,5]**. If we determine under what condition it is permissible not to take into account the effect of the resistance of the network or, the resistance of the load, this will greatly simplify the analyses of the FC and their relation to the place of the single-phase to-earth fault **[4].** Secondly, even for a single power transmission, it is possible to obtain different FCs (which is good, since additional possibilities for identifying the place of the fault emerges), because unlike electrical machines different measuring points will be available. Thus, a measuring voltage transformer is always installed on the switch-gear buses, which has two secondary windings: one assembled according to the (star with zero scheme) and the second, an (open delta), and current transformers of zero sequence are also installed (for microprocessor protection). On the side of 0.4kV step-down substations, it is also possible to perform measurements, but only the positive and negative sequence of current and voltage, since the single-phase-to-earth fault itself it considered only in a network with an isolated neutral of higher nominal voltages **[1,6]**. As you know, the zero sequence currents and voltages on the 0.4kV side does not transforms. It is also possible to use on the high side of consumer substation the same voltage transformers as on the switch-gear buses, which reveals additional opportunities for the use of FCs. As a consequence of the above, it is possible to obtain information not only on phase voltages and phase currents, but also separately on the zero-sequence component of currents and voltages [7,11]. Modern microprocessor devices allow you to separate the positive and negative sequence component from the measured signal, which again can be used to identify the place of damage on the line. Finally, it is necessary to abandon generally accepted equivalent circuits for overhead transmission lines of 11-33kV as a concentrated parameters and proceed to the description of the overhead transmission line based on theory of long lines [9], then, in the region of several kilohertz, the wave properties of the line begin to appear.

2. The mode of symmetrical switching of the higher harmonics source

In order to identify the features of the frequency characteristics FC of the overhead transmission lines in relation to the frequencies at which the current and voltage values are amplified, to assess the effect on the FC of the internal resistance of the supply network of higher voltage class, as well as the influence of the load on the step-down substations, we assume that the higher harmonic source is connected to the overhead line symmetrically for each phase, and the line itself is single supplied by low voltage source buses. In this case, suppose that the overhead line conductors are located symmetrically on the supports [2]. Thus, consider the schematic diagram of a three-phase power transmission presented in figure 1. Wherein the voltage source $e(n)$ generally has an arbitrary frequency of multiplicity *n* with respect to the industrial frequency.
The step-down substation with the transformer T is connected to the line with length *l* through the switch "S". The equivalent circuit is shown in figure 2. At high frequencies, we neglect the influence of the resistance on the frequency corresponding to the maximum voltage at the end of the supply.

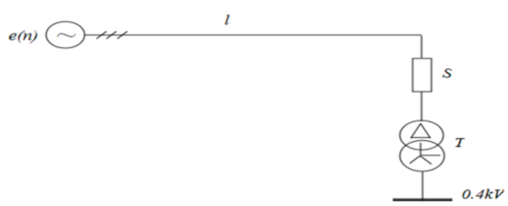

Fig.1 Electrical circuit for 6-10 kV Supply

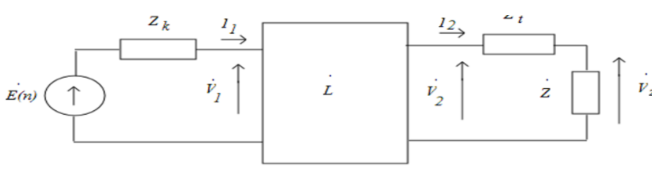

Fig.2 The equivalent circuit of the electrical supply

From the equivalent circuit figure 2., it is obvious that the maximum voltage V_2 will correspond the maximum voltage V_z which is measured on the 0.4kV side. The latter circumstance is important, because it allows the use of low- voltage measuring circuits without installing a high voltage measuring transformer at a step-down substation, which is not included in the typical schemes of substations with a voltage of 11/0.4kV [7]. At the same time, the frequency properties of the 11kV power transmission will be monitored on the 0.4kV side.
Let's write down simplified equations [1] for long distance transmission

$$\dot{V}_1 = cos(n\beta l)\dot{V}_2 + j\dot{z}_c sin(n\beta l)\dot{I}_2, \quad (1)$$

$$\dot{I}_1 = cos(n\beta l)\dot{I}_2 + \frac{J}{\dot{z}_c} sin(n\beta l)\dot{V}_2 \quad (2)$$

Where $\beta -$ is the phase change coefficient at the industrial frequency of the network,
$\dot{z}_c$ – the wave impedance of the line

The wave impedance and the wave propagation coefficients generally depend on the frequency of the current ω , because it is determined by the following expression

$$\dot{z}_c = \sqrt{\frac{r_0+\omega L_0}{g_0+\omega C_0}} \quad (3)$$

$$\dot{\gamma} = \sqrt{(r_0 + \omega L_0)(g_0 + \omega C_0)} = \alpha + j\beta \quad (4)$$

Where r_0, g_0, L_0, C_{0-} linear parameters of the overhead lines (resistance, conductance, inductance, capacitance).

For lines with voltages 11-33kV, the linear conductivity g_0 is negligible. At the frequency of $\omega_C = 50Hz$, the linear resistance r_0 of the same order with the inductive reactance $x_0 = \omega_C L_0$. But if the frequency increases by 10 times or more, then the wave impedance and the phase change coefficient can be determined in the same way as for the line without losses, i.e. according to the formulas [2]:

$$z_c = \sqrt{L_0/C_0}, \beta_\omega = \sqrt{\omega\, L_0 \omega C_0} = \omega\sqrt{L_0 C_o} = \omega\beta \quad (5)$$

For source and load impedances at high frequencies (kilohertz or more), only their inductive components can also be taken into account. Then

$$z_k = jn\omega_C L_k = jnx_k \quad (6)$$

Let's connect the parameters of the beginning and the end of the transmission, for which we give following equations

$$\dot{V}_1 = \dot{E} - jnx_k\dot{I}_1 \quad (7)$$

$$\dot{V}_2 = (jnx_t + jnx)\dot{I}_2 = jnx_n\dot{I}_2 \quad (8)$$

Combining equations (1), (2), (7), (8) we get

$$\dot{V}_1 = cos(n\beta l)\dot{V}_2 + \frac{z_C}{nx_n} sin(n\beta l)\dot{V}_2 \quad (9)$$

$$\frac{E-\dot{V}_1}{nx_k} = \frac{cos(n\beta l)}{nx_k}\dot{V}_2 - \frac{1}{z_C} sin(n\beta l)\dot{V}_2 \quad (10)$$

The equations (9) and (10) allows to determine the frequency characteristics of the electrical transmission,

$$\dot{W}_2 = \frac{\dot{V}_2}{\dot{E}} = \dot{W}_2 = \frac{1}{\left(1+\frac{x_k}{x_n}\right)cos(n\beta l) - \left(\frac{nx_k}{z_C} - \frac{z_C}{nx_n}\right)sin(n\beta l)} \quad (11)$$

The maximum value of the voltage at the end of transmission corresponds to the frequency found from the equation

$$tg(n\beta l) = \frac{1 + {x_k}/{x_n}}{{nx_k}/{z_C} - {z_C}/{nx_n}} \quad (12)$$

Note that the voltage V_z will be maximum when the voltage V_2 reaches the greatest value Fig.2. Since the analytical expression for the transfer function W_2 are somewhat simpler than if they were written relative to W_z, we will continue to operate with the magnitude of the voltage V_2, implying that in fact the measurement is made from the winding side of the lower voltage of transformer, i.e. the voltage V_z [8].
Consider a special case when the load at the end of the power transmission is turned off ($Z_n \to \infty$). In this mode, we will evaluate the effect of the internal impedance of the source on the frequency characteristics of the power transmission.
Equation (12) in this case takes the form

$$tg(n\beta l) = \frac{Z_C}{nx_k} \quad (13)$$

Equation (13) is non-linear and can be solved with respect to *n* only in numerical form.

3. Influence of system impedance and the load on "resonant" frequency

Let's evaluate the influence of the parameters listed in the title on a numerical example. Let's supply voltage 10kV with the line length of 30 km made by AC-95/16 conductors, have the following parameters:
Source: $r_e = 0.01\Omega$, $L_e = 0.01\ H$
Transmission Line: $r_0 = 0.306\ \Omega/km$, $L_0 = 1.13 \times 10^{-3}\ H/km$, $C_0 = 9.756 \times 10^{-9}\ F/km$
Wave parameters defines as for a line without losses (this is permissible, since the minimum frequency range, where we should expect "resonant" increasing in voltage, not lower than 1kHz, we are not interested in lower frequencies):

$$Z_C = \sqrt{L_0/C_0} = 10^3\sqrt{1.13/9.756} = 347.8\ \Omega$$

$$\beta = \omega_C\sqrt{L_0C_0} = 314.16 \times 10^{-6}\sqrt{1.13 \times 9.756} = 10.659 \times 10^{-4}\ {l}/{km}$$

Equation (13) takes the following form

$$tg(n \times 0.031977) = \frac{347.8}{n \times 3.1416} = \frac{110.708}{n}$$

Its solution $n = 38.5$, which corresponds to a frequency of 1925 Hz.

To assess the accuracy of the calculation performed for the power transmission in question, its model was made in the Simulink Package [10], Fig.3. The overhead line was introduced into the model as an object with distributed parameters.

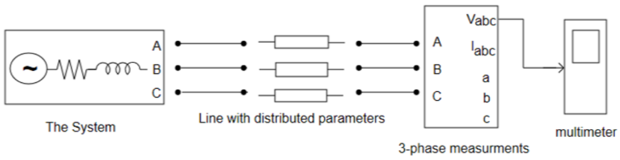

Fig.3 The power line model considering impedance of the supply network

On the adopted model Fig.3. the frequency response of the transmission has been calculated. The e.m.f. of the source was adopted to be equal to 10kV (line voltage), the frequency varied from 50Hz to 4 kHz. Voltage measurement were carried out at the open end of the overhead transmission line. Fig.4. shows the obtained frequency characteristic [10]. It can be seen that at the "resonant" frequency the voltage increased by more than an order of magnitude and the "resonant" frequency itself was 1895 Hz, which shows that the error in the calculations is less than 1.6%. For comparison, lets determine the frequency at which the voltage at the end of overhead transmission line reaches maximum, if the resistance of the source is zero.

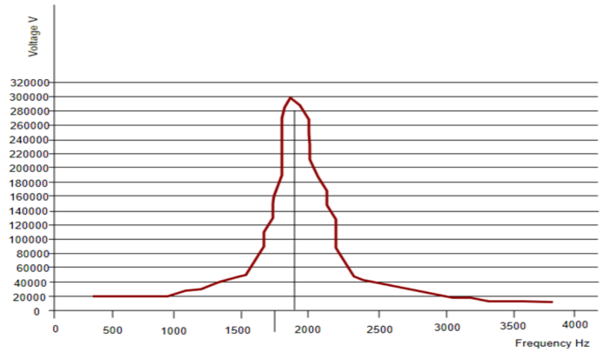

Fig.4 Frequency Characteristics of power transmission

In this case equation (13) becomes the following

$$n\beta l = \frac{\pi}{2} \quad (14)$$

From here we get for received initial data $n_r = 49.12$, which corresponds to a frequency of 2456 Hz. We conclude that for the accepted value of the internal resistance of the supply network, it is fundamentally necessary to take into account the internal resistance of the source when determining the “resonance” frequency [6]. However, it should be noted that the inductance of 0.01H corresponds to a short-circuit current on the supply buses equal to 1.84kA i.e., a fairly small current. Therefore, it is legitimate to consider how the result will change if the short-circuit current increases.

Figure 5. represents a structural diagram of a power transmission model on which a frequency characteristic FC was obtained for comparing the results of calculating “resonant” frequencies according to an idealized mathematical model of overhead transmission lines [10].

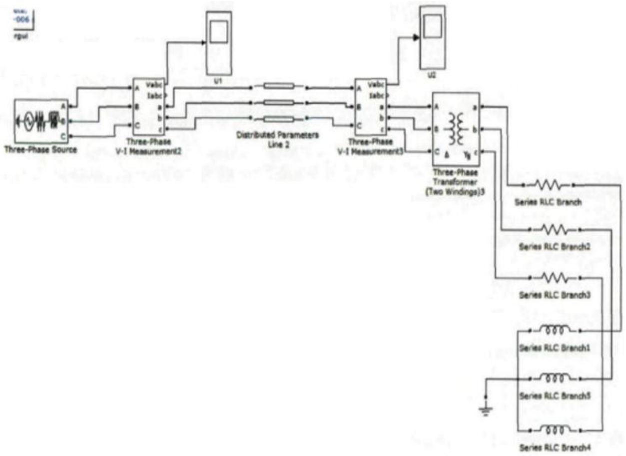

Fig.5 A structural diagram of power transmission line

Let’s assess the impact of the load. For that, let us neglect the internal impedance of the source. Then formula (12) is transformed into the following

$$tg(n\beta l) = -{}^{nx_n}/_{Z_c} \quad (15)$$

We will evaluate the effect of the load and length of the overhead lines on the resonant in an example of a particular power transmission. We introduce the following notations: $\mu = {}^{x_n}/_{Z_c}$; The range of change μ is defined as follows. Let load coefficient of the transformer T (fig.2) equal to $k_l = 0.7$; the tangent of the load angle is $tg\varphi_l = 0.4$ The voltage at the transformer inputs $V = 6kV$. Nominal voltage of the transformer is $V_n = 6.3\ kV$, Rated KVA of the transformer is $S_n = 1250\ kVA$. The short-circuit voltage of the transformer $V_{sc} = 5\%$, then

The reactance of the transformer

$$x_T = \frac{V_{sc}}{100}\frac{V_n^2}{S_n} = 0.05\ \frac{6.3^2}{1.25} = 1.59\ \Omega;$$

Load reactance

$$x = \frac{V^2}{K_l S_l} sin\varphi_l = \frac{6^2}{0.7\times1.25} 0.371 = 15.26\Omega$$

Since $z_c = 347.8\ \Omega$, then

$$\mu = {}^{(1.58\ + 15.28)}/_{347.8} = 0.0485\ p.u$$

For network of rated voltage of 11 kV with the rest of the same initial data we have $\mu = 0.147\ p.u.$

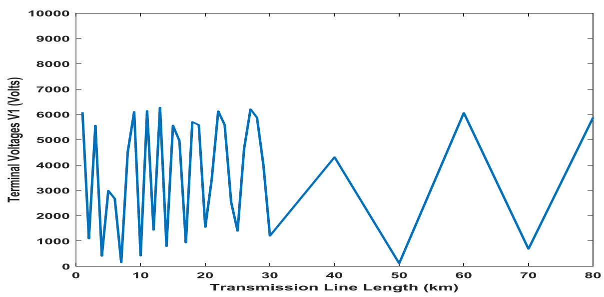

Fig.6 Relationship between terminal voltage versus transmission line length

It should be demonstrated a Frequency Characteristics taking into account the loads obtained on the model, Fig. 5

It is accepted that for the load R = 1.93 Ohm, L = 0.000195 H, a transformer with a capacity of 630 kVA

Consider the combined effect of the load and resistance of the system on the "resonant" frequencies. The patterns of change in the "resonant" frequency corresponding to the values of the function y = 0 obtained from the complete equation (12) which would have the following form

$$y = tg(n\beta l) - \frac{1 + {}^{x_k}/_{x_n}}{{}^{nx_k}/_{Z_{\tilde{n}}} - {}^{1}/_{n\mu}}$$

The curves are constructed when the length of the line is 30 km;

$$Z_c = 347.8\ \Omega\ , \beta = 0.0010659\frac{rad}{km}\ ;$$
$$1 - \ I_k = \infty(x_k = 0), \mu = 0{,}0485;$$
$$2 - \ I_k = 26.24\ kA, \mu = 0.0485;$$
$$3 - I_k = 26.24kA\ , \mu = 0.0970\ ;$$
$$4 - I_k = \infty(x_k = 0), \mu = 0.1455\ ;$$
$$5 - I_k = 5.25kA\ \ , \mu = 0.485.;$$
$$6 - I_k = 5.25kA\ , \mu = 0.485.$$

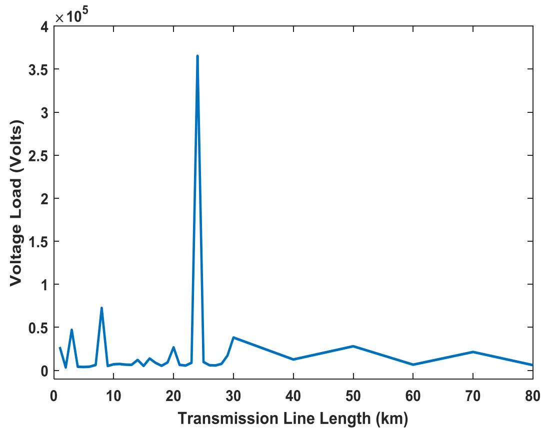

Fig.7 Relationship between voltage load (p.u) versus transmission line length

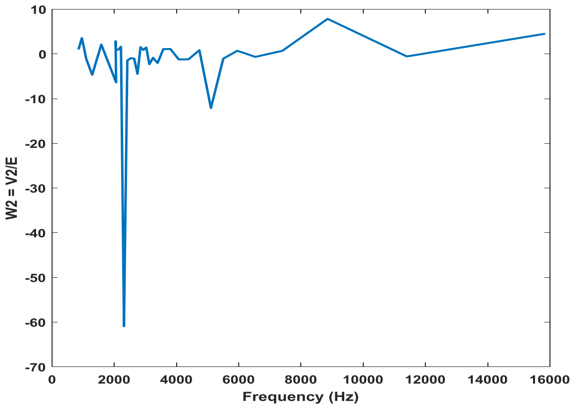

Fig.8 Relationship between W2 versus frequency

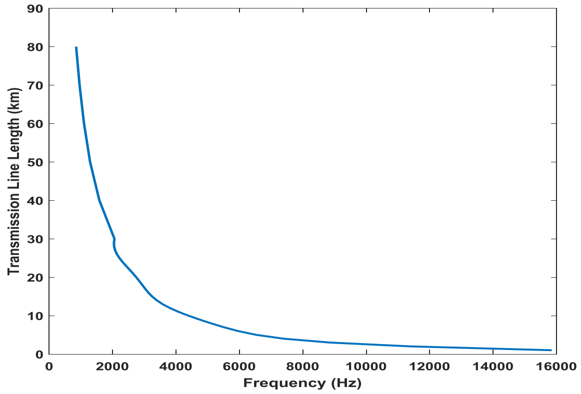

Fig.9 Dependance of transmission line lengths and frequency

The resistance of the power grid has a negligible effect on the "resonant" frequency, which allows the use of graphs constructed assuming an "infinite" value of the short circuit current on the buses.

Conclusions

When constructing frequency responses for electrical distribution networks with a voltage of 11 - 33 kV, it is necessary to use mathematical models of power lines, represented as long lines. Using the assumption that it is possible to transition from the general equations of the long line to the partial ones, describing the line without loss of active power, facilitates calculations and provides acceptable accuracy at frequencies greater than 1 kHz (error within a few percent).

Power transmissions of the considered class of voltages show "resonant" properties at frequencies ranging from several tens of kilohertz, which makes it possible to build up from the higher harmonics generated by the industrial and household load of rural consumers. Signal amplification in the region of high frequencies is determined by the wave properties of power transmission. Frequency responses can be removed relative to the beginning and end of the transmission

Increasing the level of short-circuit currents on the buses of the power center leads to the upsetting the resonant peak of the frequency response signal relative to the beginning of the line, which makes it preferable to use frequency characteristics relative to the end of the transmission. For the frequency characteristics of the voltages of the zero sequence, it is possible to remove them only relative to the beginning of the power transmission, where zero sequence voltage transformers are installed on the 11-33 kV buses of the power center. In this case, combinations of network parameters and the magnitude of the contact resistance are possible, when the "resonant" amplification of higher harmonics is not evident.

Appendix

No.	Transmission Line Length (km)	Frequency (Hz)
1	1	15848
2	2	11402
3	3	8855
4	4	7401
5	5	6533
6	6	5954
7	7	5503
8	8	5105
9	9	4735
10	10	4389
11	11	4075
12	12	3803
13	13	3575
14	14	3391
15	15	3245
16	16	3127
17	17	3027
18	18	2935
19	19	2842
20	20	2745
21	21	2640
22	22	2530
23	23	2419
24	24	2311
25	25	2215
26	26	2136
27	27	2079
28	28	2048
29	29	2042
30	30	2058
31	40	1582
32	50	1289
33	60	1098
34	70	958
35	80	850

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