

The Limited Pressure Model In A Thin-Walled Circular Plate, Based On A Mathematical Model With Limited Plastic Deformation.

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Abstract:

An approximated mathematical model for uniform internal pressure is studied, theoretically based on an ideal elastoplastic deformation theory and based on an equal divided thin walled circular shell, which numerically solved and used the results for testing experimentally and simulate by NX 8 Siemens finite element software under the resulted stress constrain criteria in yield stress and more 5% of it, with a limit of plastic deformation appearance in the model selected for two different material Steel 1006 and Aluminum 6053, which were under An investigation of bending test ,for estimations the resulted displacements values during elastoplastic deformation. The using of getting model given an error percentage compared with the basic theoretical mathematical equation of internal uniform pressure , which given 1.5-2% for Steel 1006 and 0.1-0.19% for Aluminum 6053, which is supported the correctness and the acceptance of using the suggested getting approximated mathematical model in this study.

Keywords: thin plate & shell theory, ideal elastoplastic deformation theory, finite element method

نموذج ضغط محدود في لوحة دائرية رقيقة
الجدران مستند إلى نموذج رياضي مع تشوه لدن محدود

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المقدمة

موديل رياضي تقريبي لضغط داخلي متساوي التأثير تم دراسته، ونظريا تم بناءه على اساس النظرية المثالية للتشوه المرن-اللدن وتم بناءه على اساس تقسيم جدار رقيق السمك لصفحة دائرية الشكل لجزيئين متساويين ، والتي تم حلها عدديا واستخدام النتائج لغرض الفحص العملي ولغرض استخدامها في برنامج محاكاة ان اكس ٨ سيمنس للتحليل باستخدام العناصر المحددة تحت محددات الاجهادات الناتجة ضمن اجهاد الخضوع واجهادات اكثر منه بنسبة ٥% مع محدد لظهور التشوه اللدن في الموديل المختار لمادتين مختلفتين هما الستيل ١٠٠٦ والالمنيوم ٦٠٥٣ واللدان خضعا للاختبار بفحص الانحناء لحساب الازاحات الناتجة من التشوه في مرحلة التشوه المرن-اللدن . ان استخدام الموديل الناتج اظهر نسبة خطأ بالمقارنة مع المعادلة الرياضية النظرية الاساسية للضغط الداخلي والتي اعطت نسبة خطأ ١،٥-٢% في حالة الستيل ١٠٠٦ و ٠،١-٠،١٩،٠٠% في حالة الالمنيوم ٦٠٥٣ وهذا ما عزز دقة وقبول الموديل الرياضي التقريبي الناتج المقترح في هذا البحث.

كلمات البحث: نظرية الصفائح والهياكل رقيقة الجدران، ونظرية التشوه اللدن المثالية، طريقة العناصر المحددة

1.Introduction

In thin-walled shells usually is not permitted occurrence of plastic deformation in the thickness and the conditions is used for the calculation of shell strength only elastic deformations, and the development of calculation algorithms in structural project is impelled by a constant challenge in the search of more accurate and fast design tools in engineering [1]. The result is a system of ordinary differential equations where the solution is analytic after evaluation of mathematical modeling . The transverse displacement presents important dependence on the shell thickness vs radius, as the shell can be a thin-walled one up to a moderately thick one, where the surface displacement ranges until the extreme edges, which is not simple case analyzed, and a comparison with finite element methods is usually presented [2]. The construction material is compressible, and the normal to the middle surface Ω stress σ_3 are small compared with the rest of the stresses $\sigma_1, \sigma_2, \sigma_{13}$. As is well known, in this case, the relationship between stress $\sigma_1, \sigma_2, \sigma_{13}$ and deformations are recorded as follows of set of equations : [3]

$$\begin{aligned} \sigma_1 &= \frac{E}{1-\nu^2}(\varepsilon_1 + \nu \varepsilon_2) + \frac{E}{1+\nu}[-\varpi \varepsilon_1 + f_T \cdot (\varepsilon_1 + \varepsilon_2)]; \\ \sigma_2 &= \frac{E}{1-\nu^2}(\varepsilon_1 + \nu \varepsilon_2) + \frac{E}{1+\nu}[-\varpi \varepsilon_1 + f_T \cdot (\varepsilon_1 + \varepsilon_2)]; \sigma_{13} = \frac{3 Q_1}{2 h} \left[1 - \left(\frac{2z}{s} \right)^2 \right]; \\ \sigma_i &= 3G(1-\varpi) \varepsilon_i; \sigma_i = \sqrt{\sigma_1^2 + \sigma_2^2 - \sigma_1 \sigma_2 + 3\sigma_{13}^2}; \varepsilon_i = \frac{2}{3} \sqrt{(\varepsilon_1 + \varepsilon_2)^2 (1 + \alpha_1 + \alpha_1^2) - 3\varepsilon_1 \varepsilon_2}; \\ \alpha_1 &= \frac{3\nu + (1-2\nu)\varpi}{3(1-\nu) - 2(1-2\nu)\varpi}; f_T = \frac{\varpi \left(\frac{1-4\nu+\nu^2}{1-\nu} \right) - (1-2\nu)\varpi}{3(1-\nu) - 2(1-2\nu)\varpi}. \end{aligned} \tag{1}$$

When the solution of problem needs to describe the elastic-plastic deformation of the shell using the linear hardening law, according to a part of equation (1), the function of plasticity is defined as in Fig.(1), that suppose 5% over the value of yield stress value in elastic zone:[4].

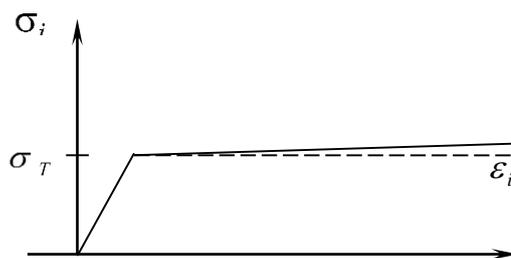


Fig.(1) Theory of perfectly plasticity relation

The forces and moments are expressed in terms of the components of deformation from the known formulas:[5]

$$T_1 = B[\varepsilon_1 + \nu \varepsilon_2] + \Delta T_1; \quad M_1 = D(\kappa_1 + \nu \kappa_2) + \Delta M_1; \quad T_2 = B[\varepsilon_2 + \nu \varepsilon_1] + \Delta T_2; \\ M_2 = D(\kappa_2 + \nu \kappa_1) + \Delta M_2; \tag{2}$$

Where $B = Eh / (1 - \nu^2)$; $D = Eh^3 / [12(1 - \nu^2)]$.

To determine the forces and moments using equilibrium equations fragment design :[6].

$$\Delta T_1 = 2G \int_{-h/2}^{h/2} [-\varpi \varepsilon_1 + f_T \cdot (\varepsilon_1 + \varepsilon_2)] dz; \quad \Delta M_1 = 2G \int_{-h/2}^{h/2} [-\varpi \varepsilon_1 + f_T (\varepsilon_1 + \varepsilon_2)] z dz; \\ \Delta T_2 = 2G \int_{-h/2}^{h/2} [-\varpi \varepsilon_2 + f_T \cdot (\varepsilon_1 + \varepsilon_2)] dz; \quad \Delta M_2 = 2G \int_{-h/2}^{h/2} [-\varpi \varepsilon_2 + f_T (\varepsilon_1 + \varepsilon_2)] z dz. \tag{3}$$

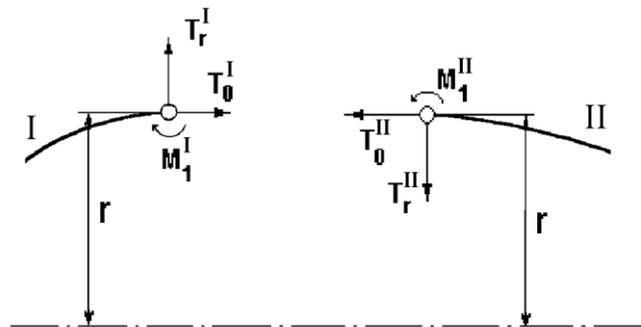


Fig. (2). Forces and moments in the docking section

In those sections where docked the shell in Fig.(2), after the effecting pressure, which satisfy the conditions of continuity of the rotation angles and displacements, the equilibrium equation (4) can be written for each of the parts, and for joining the fragments of shells used coupling conditions. [7]

$\varpi_1^I = \varpi_1^{II}$, $u_0^I = u_0^{II}$, $u_r^I = u_r^{II}$, as well as the conditions for forces and moments:

$$T_0^{II} - T_0^I = 0, \quad T_r^{II} - T_r^I = 0, \quad M_1^{II} - M_1^I = 0. \tag{4}$$

And the vector is introduced $Y = (T_1, Q_1, M_1, u, w, \varpi_1)^T$, defined in sections of the shell with coordinates s_i , $i = \overline{1, n}$, whose components are unknown force T_1, Q_1 , moment M_1 ,

displacement u, w and a rotation angle ϖ_1 . The system of differential equations is solved numerically using the method of Parallel sweep algorithm [8].

The model is completed by calculate the stress strain state ,Under principle of the theory of rigid-plastic deformation of bodies, for any continuous medium , the following basic energy equation is used [4] :

$$T(\sigma_{ij}, \xi_{ij}, \bar{U}) = \iiint_V Po(\sigma_{ij}, \xi_{ij}) dV - \iint_{\Omega} \bar{P}\bar{U} d\Omega = 0. \tag{5}$$

With respect to the calculation of the limit load with elastic-plastic axisymmetric deformation of shells As in [9], the power density plastic deformations in plane stress is written as the potential energy equation Po as follows:

$$Po = \sigma_{11}\xi_{11} + \sigma_{22}\xi_{22} + \sigma_{12}\xi_{12} . \tag{6}$$

For an ideally rigid-plastic material with the law of Saint-Venant ,and the relationship between stress and strain rate is determined by the following formulas:

$$\sigma_{11} = \frac{2}{3} \frac{\sigma_T}{\xi_i} (2\xi_{11} + \xi_{22}), \sigma_{22} = \frac{2}{3} \frac{\sigma_T}{\xi_i} (2\xi_{22} + \xi_{11}), \sigma_{12} = \frac{1}{3} \frac{\sigma_T}{\xi_i} \xi_{12}. \tag{7}$$

$$\xi_i = \frac{2}{\sqrt{3}} \sqrt{\xi_{11}^2 + \xi_{22}^2 + \xi_{11}\xi_{22} + 0.25\xi_{12}^2} . \tag{8}$$

2.Methodology

Given the equilibrium equation (3) for the shell fragment Fig.(2), and other relations presented here, using known mathematical transformations , for the vector of unknown after load has been effected, the equations is written as follows:

$$\frac{dY}{ds} = A(s)Y + F(s, Y) + C(s), \tag{9}$$

Where

$$A(s) = \begin{pmatrix} (\nu - 1)\psi & -k_1 & 0 & B_0\psi^2 & B_0\psi k_2 & 0 \\ k_1 + \nu k_2 & -\psi & 0 & B_0\psi k_2 & B_0k_2^2 & 0 \\ 0 & 1 & (\nu - 1)\psi & 0 & 0 & D_0\psi^2 \\ \frac{1}{B} & 0 & 0 & -\nu\psi & -k_1 - \nu k_2 & 0 \\ 0 & 0 & 0 & k_1 & 0 & -1 \\ 0 & 0 & \frac{1}{D} & 0 & 0 & -\nu\psi \end{pmatrix},$$

$$B_0 = B(1 - \nu^2), D_0 = D(1 - \nu^2), \psi = \cos \theta / r,$$

$$F = \begin{pmatrix} \psi(\Delta T_2 - \nu \Delta T_1) \\ k_2(\Delta T_2 - \nu \Delta T_1) \\ \psi(\Delta M_2 - \nu \Delta M_1) \\ -\frac{1}{B} \Delta T_1 \\ 0 \\ -\frac{1}{D} \Delta M_1 \end{pmatrix}, \quad C = \begin{pmatrix} -X_1 \\ -X_3 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}.$$

As a result, a system is used the equilibrium equations of the following form:

$$\frac{dT_1}{ds} = 0; \quad \frac{dM_1}{ds} = 0; \quad \frac{dw}{ds} = 0;$$

$$\frac{dQ_1}{ds} = k_1 T_1 - \frac{X_3}{2}; \quad \frac{du}{ds} = \frac{1}{B(1+\nu)}(T_1 - \Delta T_1) - k_1 w; \quad \frac{dw_1}{ds} = \frac{1}{D(1+\nu)}(M_1 - \Delta M_1). \quad (10)$$

To calculate the stress-strain state structure, need to solve a system of ordinary differential equation (9) and equation (10) subject to the conditions of conjugation the shell fragments in equation (4), and all the boundary conditions and the system of differential equation (5) is solved numerically using the method of Parallel sweep algorithm. Consistently determine the approach to the solution of equation (9) - vector $Y^{(k)}$ ($k = 1, 2, 3, \dots$). $Y^{(k)}$ to $Y^{(k+1)}$ in the non-linear part of the equation (5) are substituted known values $Y^{(k)}$ and solve linear boundary value problem for the system of equations:

$$\frac{dY^{(k+1)}}{ds} = A(s)Y^{(k+1)} + F(s, Y^{(k)}) + C(s). \quad (11)$$

The completed model need to include the stress strain state, which solve through the potential energy relation related with change of model state, starting with substituting equation (7) into equation (6), equation (12) is obtained:

$$P_0 = \frac{4}{3} \frac{\sigma_T}{\xi_i} (\xi_{11}^2 + \xi_{22}^2 + \xi_{11}\xi_{22} + 0.25\xi_{12}^2) = \sigma_T \xi_i. \quad (12)$$

For shells expression is written as follows:

$$\xi_i = \frac{2}{\sqrt{3}} \sqrt{P_\epsilon + 2zP_{\epsilon\kappa} + z^2P_\kappa}. \quad (13)$$

Where $P_\epsilon = \dot{\epsilon}_{11}^2 + \dot{\epsilon}_{22}^2 + \dot{\epsilon}_{11}\dot{\epsilon}_{22} + \frac{1}{4}\dot{\epsilon}_{12}^2$; $P_\kappa = \dot{\kappa}_{11}^2 + \dot{\kappa}_{22}^2 + \dot{\kappa}_{11}\dot{\kappa}_{22} + \frac{1}{4}\dot{\kappa}_{12}^2$;

$$P_{\epsilon\kappa} = \dot{\epsilon}_{11}\dot{\kappa}_{11} + \dot{\epsilon}_{22}\dot{\kappa}_{22} + \frac{1}{2}\dot{\epsilon}_{11}\dot{\kappa}_{22} + \frac{1}{2}\dot{\epsilon}_{22}\dot{\kappa}_{11} + \frac{1}{4}\dot{\epsilon}_{12}\dot{\kappa}_{12}. \quad (14)$$

In the equation (14) $\dot{\epsilon}_{11}, \dot{\epsilon}_{22}, \dot{\epsilon}_{12}$ - strain rate and shear elongation middle surface; $\dot{\kappa}_{11}, \dot{\kappa}_{22}, \dot{\kappa}_{12}$ - strain rate of change of curvature of the middle surface of the shell. For the upper limit of load

for shells of revolution used the following algorithm the relations defining the stress-strain state of the shell for a sequence of values possible component of displacement were taken values:

$$\dot{u}^* = \frac{u_{\kappa+1} - u_{\kappa}}{\Delta t}, \quad \dot{w}^* = \frac{w_{\kappa+1} - w_{\kappa}}{\Delta t}, \quad \kappa = 1, 2, 3, \dots \quad (15)$$

$t_{\kappa} = t_{\kappa-1} + \Delta t$, $\kappa = 1, 2, 3, \dots$, where Δt - step in the parameter t . Consequently, for each of the load values $P_{\kappa}(t_{\kappa})$ components were moving points Ω of the shell surface u_{κ}, w_{κ} .

As viewed axially symmetric deformation shell, then $\dot{\epsilon}_{12} = 0$; $\dot{\kappa}_{12} = 0$. Using these notations in this relations equation (5), equation (15) for kinematically possible strain rates shell, the following expressions are obtained with sign * to indicate the movement of calculations steps on the points of surface :

$$\dot{\epsilon}_{11}^* = \frac{d\dot{u}^*}{ds}, \quad \dot{\epsilon}_{22}^* = \frac{\cos \theta}{r} \dot{w}^*, \quad \dot{\kappa}_{11}^* = \frac{d\dot{\omega}_{11}^*}{ds}, \quad \dot{\kappa}_{22}^* = -\frac{\cos \theta}{r} \frac{d\dot{w}^*}{ds}. \quad (16)$$

Using the set of equations (12 – 16), the following expression is obtained:

$$P_{\kappa}^* = \sigma_T \xi_i^*. \quad (17)$$

As noted previously, the action on the shell considered uniformly distributed load perpendicular to the surface Ω . In this case, for each load value $P_{\kappa}(t_{\kappa})$

$$\iint_{\Omega} \bar{P} \bar{U}^* d\Omega = \iint_{\Omega} P_{\kappa} \dot{w}^* d\Omega = P_{\kappa} \iint_{\Omega} \dot{w}^* d\Omega. \quad (18)$$

In view of equation (5), equations (17-18), a new equation can be obtained with consider of equivalent stress should less than the yield stress in perfect plastic stress –strain relations Fig.(1).

$\sigma_T \iiint_V \xi_i^* dV \geq P_{\kappa} \iint_{\Omega} \dot{w}^* d\Omega$. Consequently find :

$$P_{\kappa} \leq \frac{\sigma_T \iiint_V \xi_i^* dV}{\iint_{\Omega} \dot{w}^* d\Omega}. \quad (19)$$

If used for the calculation of values ξ_i^*, \dot{w}^* , The corresponding limit of the shell, then found by the equation (19), gives the value P_{ef} will be equal to the pressure effected in the shell in which there is a limit state. And it is necessary to calculate integrals $\iiint_V \xi_i^* dV, \iint_{\Omega} \dot{w}^* d\Omega$ in terms of volume V and the area of the surface of the shell Ω .

As viewed axially symmetric deformation of the shell, the integral can be represented by the integral over the length of the meridian arc s , that

$$\iint_{\Omega} \dot{w}^* d\Omega = 2\pi \int_s \dot{w}^*(s) r(s) ds . \tag{20}$$

Considering that the integral over volume of the envelope can be represented as an integral over the thickness and surface area of the middle shell, and also taking into account the transformation $(-h/2 \text{---} h/2)$ of the form equation (20), can be obtained equation (21).

$$\iiint_V \xi_i^* dV = \int_{-h/2}^{h/2} \iint_{\Omega} \xi_i^* d\Omega dz = 2\pi \int_{-h/2}^{h/2} \int_s \xi_i^*(s, z) r(s) ds dz . \tag{21}$$

Substituting the expressions of equation (20) and equation (21) into equation (19), a new equation(22) is Formed:

$$P_{ef} = \frac{\sigma_T \int_{-h/2}^{h/2} \int_s \xi_i^*(s, z) r(s) ds dz}{\int_s \dot{w}^*(s) r(s) ds} . \tag{22}$$

Integrals in equation (22) is computed numerically , chosen N section defines the coordinates of the arc length of meridian S_n , $n = \overline{1, N}$, in these cross sections through the thickness of the shell along the z –axis through the first and second layer , perpendicular to the middle surface, a step Δz_{κ} chosen K points defined by coordinates Z_{κ} ($\kappa = \overline{1, K}$). Consequently, the volume of the shell Selected $N \times K$ points $M(S_n, Z_{\kappa})$. These points are used for numerical computation in equation (22) integrals.

$$\int_{-h/2}^{h/2} \int_s \xi_i^*(s, z) r(s) ds dz = \sum_{n=1}^N \sum_{\kappa=1}^K \xi_i^*(s_n, z_{\kappa}) r(s_n) \Delta s_n \Delta z_{\kappa} ;$$

$$\int_s \dot{w}^*(s) r(s) ds = \sum_{n=1}^N \dot{w}^*(s_n) r(s_n) \Delta s_n . \tag{23}$$

Thus, substituting equation (23) into equation (22), the solved model as a new equation is Formed:

$$P_{ef} = \frac{\sigma_T \sum_{n=1}^N \sum_{\kappa=1}^K \xi_i^*(s_n, z_{\kappa}) r(s_n) \Delta s_n \Delta z_{\kappa}}{\sum_{n=1}^N \dot{w}^*(s_n) r(s_n) \Delta s_n} . \tag{24}$$

This model can use for calculating the pressure in thin shell taking in to account the possibility of using the value of σ less than σ_T to calculate the value of pressure and compare with the liner solution ,then use this value equal or more than σ_T .

3.Numerical solution

The mathematical model is solved depended on finding the parameters in the all equations, which is solved consequently using Matlab R2009a. In general the numerical procedure consists of dividing the walled thin shell thickness h into two layers $h/2$, and solve it by integration from $-h/2$ to $h/2$, the loads with elastic assumption $\sigma_{eq} < \sigma_T$, is developed to elastic-plastic in first layer without getting the full plastic.

The important step is solved by updating the parameters of equations and get the maximum equivalent stress in every point on the length of arc(s), and the distribution of the stress in the elastic-plastic region until the conditions of $\sigma_{eq} \approx \sigma_T$ is achieved, then the iteration is stopped with determined the plastic point on the arc length and the maximum upper limit of the first layer, or checking for full plastic deformation in the plate according to the theoretical equation below is known in the plate shell limit load value, which had calculated by the formula [10]

$$P = 11.3 \frac{\sigma_T}{4} \frac{h^2}{R^2} . \quad (25)$$

For the comparison ,The finite element tool is giving more visualization of the model Fig.(4), the NX Siemens Software is used Fig.(5) to built the different thin walled shell models and solve it to get the visualization of distributions of deformations and stress with their values ,which can change the input parameters to get more indication on the shell structure. Table(1) shows the mechanical properties are taken from the seller company, but the nearest information about the mechanical properties are the Steel 1006 and Aluminum 6053, have compared and taken from [11]:

Table (1) the mechanical properties of tested materials.

Material	Yield strength MPa	Modulus of elasticity GPa	Density (x 1000 kg/m ³)	Poisson's Ratio	Composition & notes
Steel	285	200	7.870	0.28	C= 0.065 - 0.075
Aluminum	217	70	2.66	0.33	Al=98%

4.Experimental solution

The model is tested experimentally using the device in Fig.(3), which is made from stainless-steel under the Russian standard (GOST) for a thin walled circular shell with radius not exceed 0.1 m and with Air pressure not exceed 7 bar.

A thin plate has taken in our experiment with radius $R = 0.1$ m plate thickness $h = 0.001$ m, under internal uniformly distributed pressure P , which is solved using the model in equation (24) numerically and the pressure values are compared with theoretical values Fig.(4).

The pressure – displacement relation is recorded Fig.(5) and with NX 8 finite element software is used to simulate the model with a large dimensions, which is taken for example radius $R = 1$ m plate thickness $h = 0.01$ m, under the same conditions and values of pressure which are obtained numerically, then the comparison has taken for two different materials Steel 1006 and Aluminum 6053 for insure the correction of the model as bellow :

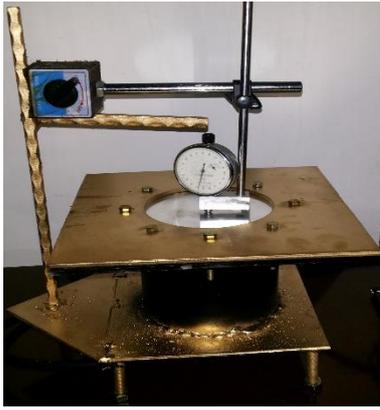


Fig. (3). Device made for testing clamped at the edges of circular plate

5.Results:

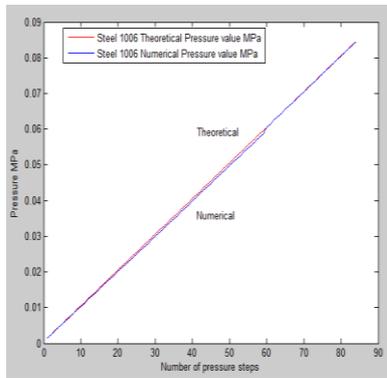


Fig.(4) Pressure compression values in plate solving numerically using Matlab R2009a

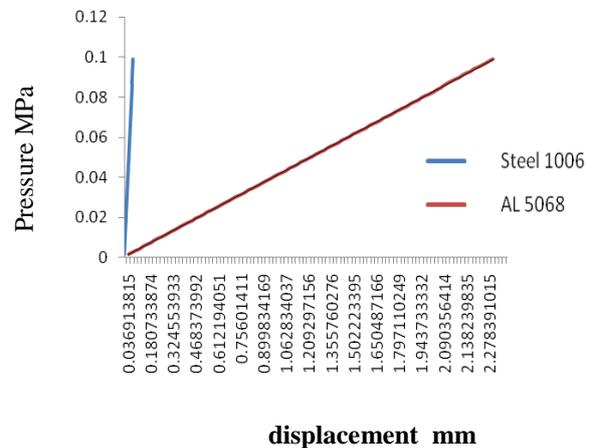


Fig.(5) Pressure-Displacements measurements in plate using pressure and dial gage

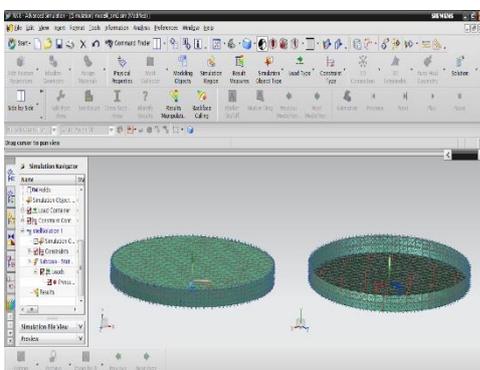


Fig. (6). Modeling the circular plate using NX8 software.

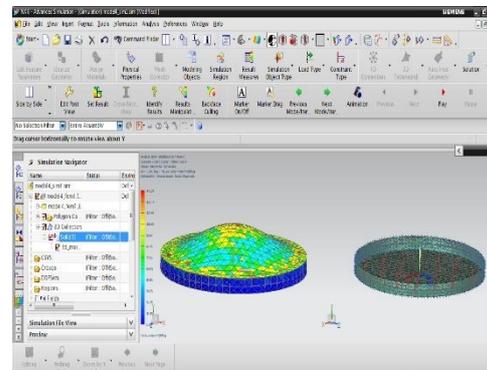


Fig. (7). Stress analysis of thin circular plate (Steel 1006)

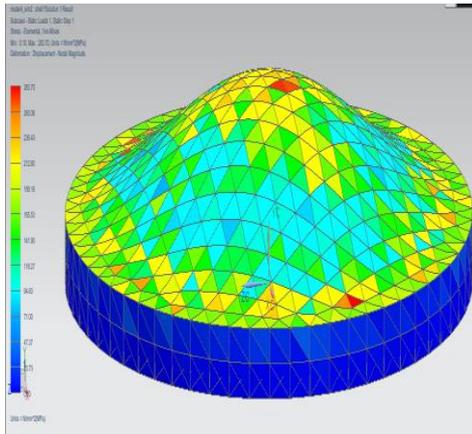


Fig. (8). σ_y Stress analysis of thin circular plate (Steel 1006)

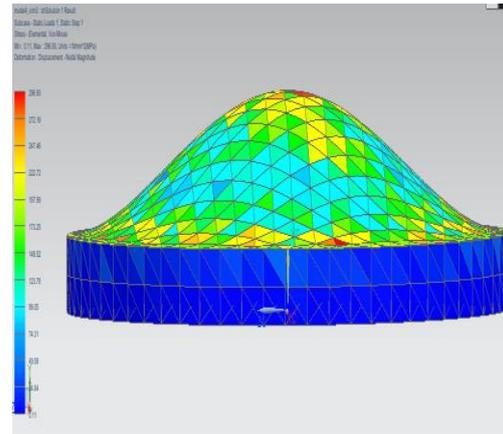


Fig. (9). σ_T Stress analysis of thin circular plate (Steel 1006)

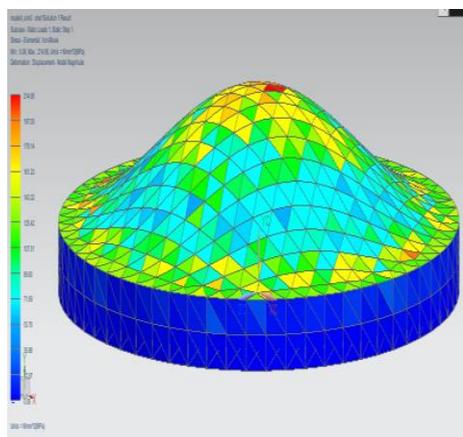


Fig. (10). σ_y Stress analysis of thin circular plate (Aluminum 6053)

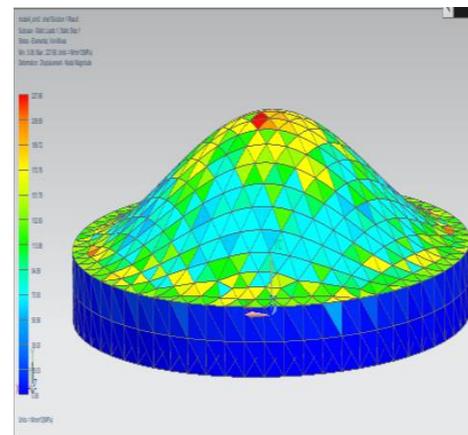


Fig. (11). σ_T Stress analysis of thin circular plate (Aluminum 6053)

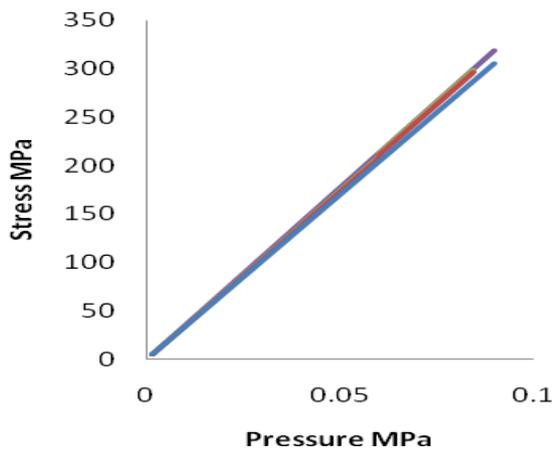


Fig.(12) Pressure-Stress compression in plate Steel 1006

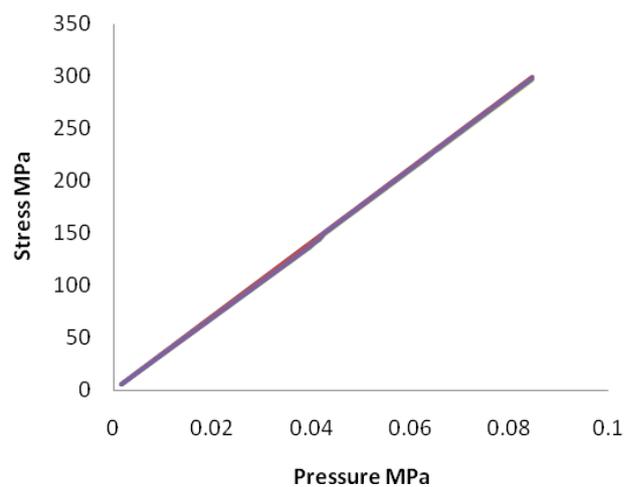


Fig.(13) Pressure-Stress compression in plate Aluminum 6053

Table (2).the results comparison between two materials with maximum stress`

teel 1006				
Pressure MPa	σ_{theory} MPa	$\sigma_{Numerical}$ MPa	σ_{SimNX} MPa	Fig. No:
0.080	285	284.81	283.70	6
0.08454	299.25	298.98	296.93	7
Aluminum 6053				
0.061	217	216.04	214.95	8
0.064	228.46	228.00	227.66	9

6. Discussion & Conclusions

- 1- The error percentage in pressure in the Steel 1006 was 1.5-2% until the yield limit stress, but the error percentage in pressure between the yield stress , and the yield_{0.05} was 0.09% where the pressure theoretically 0.0845 MPa and the pressure numerically 0.084464 MPa. The increasing in the pressure more than that values, is made the error percentage in pressure is reached to 4.5% with full plastic deformation, where the pressure theoretically 0.099 MPa and the pressure numerically 0.094545 MPa.
- 2- The error percentage in pressure in the Aluminum 6053 was 0.1-0.19% until the yield limit stress, but the error percentage in pressure between the yield stress , and the yield_{0.05} was 0.19% where the pressure theoretically 0.0645 MPa and the pressure numerically 0.0644 MPa. The increasing in pressure more than that value, the error percentage in pressure is reached to 4.5% with full plastic deformation where the pressure theoretically 0.082 MPa and the pressure numerically 0.086 MPa.
- 3- From 1 and 2 shows the stability of the getting model because of the material in elastic deformation zone and the errors indicated the correctness of the iterations in numerical solutions using Matlab R2009a, and the increasing of errors because of the effect of increasing plastic deformation appearance in the first layer and going to the second layer , which going to consider one deformed layer in some points during the numerical solutions of solving the model.
- 4- The stress analysis under the yield stress of two different material Fig.(8) and Fig.(10) are used to check the stress resulting du to the pressure calculated numerically , as well as in the yield_{0.05} in Fig.(9) and Fig.(11) respectively. These checking are calculated in finite element model using NX 8 software, which gives a good visualization results of stress analysis. The results that are compared with the practical and theoretical and numerical values to the Steel 1006 and Aluminum 6053 as shown respectively in Fig.(12) and Fig.(13). Which is showed the nearest of results between the suggested model and the experimental and the theoretical equation (25), these are supported the correctness of the suggested model (equation 24).
- 5- From table (2), in general ,the Steel 1006 error percentage in pressure is less than in Aluminum 6053, in the range of yield stress but larger in the yield_{0.05} stress, This is because of the behavior of mechanical properties and the molecular distributions of these materials during in the bending test.
- 6- In over all that indicated of correctness of the model is obtained with error percentage in pressure between 0.099-4.5% as the possibility of using this model for circular plate thin shell, which recommended of using this model only in elastoplastic zone without reaching to the full plastic deformation, and do not recommended using higher uniform internal pressure.

. 7.References

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List of symbols

No	Name	Descriptions	Unit
1	E	Modulus of elasticity	MPa
2	ν	Poisson's ratio	-----
3	G	Shear modulus	MPa
4	$\omega(s, z)$	Function of plasticity	-----
5	s	Coordinate of a point on the arc of shell appeared after deformation	-----
6	z	Coordinate of z-axis	-----
7	Q_1	Shear force	N
8	o, r	Components of surface loads along the meridian and the direction normal to the surface point Ω	-----
9	σ_1 and σ_2	The principal stresses	MPa
10	σ_T	The ideally elastic-plastic function of yield stress	MPa
11	ε_i	The ideally elastic-plastic function of strain	-----
12	ϖ_1^I	Angles of rotation on the first edge of the shell fragment.	deg
13	u_0^I	Displacement along the meridian du to effected force and moment on the first edge of the shell fragment.	m
14	u_r^I	Displacement in normal direction du to effected force and moment on the first edge of the shell fragment.	m
15	T_0^I	Force along the meridian on the first edge of the shell fragment.	N
16	T_r^I	Force in normal direction on the first edge of the shell fragment.	N
17	M_1^I	Moment on the first edge of the shell fragment.	N.m
18	ϖ_1^{II}	Angles of rotation on the second edge of the shell fragment.	deg
19	u_0^{II}	Displacement along the meridian du to effected force and moment on the second edge of the shell fragment.	m
19	u_r^{II}	Displacement in normal direction du to effected force and moment on the second edge of the shell fragment.	m
20	T_0^{II}	Force along the meridian on the second edge of the shell fragment.	N
21	T_r^{II}	Force in normal direction on the second edge of the shell fragment.	N
22	M_1^{II}	Moment on the second edge of the shell fragment.	N.m
23	ξ_{ij}	Strain in a plastic deformation coordinated point ij.	----
24	Po	Potential energy.	J
25	*	Indicate the movement of calculations steps on the points of shell surface.	----
26	P_{ef}	Pressure effected in the shell in which there is a limit state.	MPa
27	$\dot{\varepsilon}_{11}, \dot{\varepsilon}_{22}, \dot{\varepsilon}_{12}$	Strain rate and shear elongation middle surface.	-----
28	$\dot{\kappa}_{11}, \dot{\kappa}_{22}, \dot{\kappa}_{12}$	Strain rate of change of curvature of the middle surface of the shell.	-----
29	h	Thickness of the thin shell	m
30	R	Radius of the thin shell	m